Gravitational Collapse and Spacetime Singularities

PANKAJ S. JOSHI

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GRAVITATIONAL COLLAPSE AND SPACETIME SINGULARITIES

Physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star when it collapses under its own gravity at the end of its life cycle is one of the most important questions in gravitation theory and relativistic astrophysics, and is the foundation of blackhole physics.

General relativity predicts that continual gravitational collapse gives rise to a spacetime singularity, which may be hidden inside an event horizon or visible to external observers. This book investigates these issues, and shows how such visible ultra-dense regions arise naturally and generically as an outcome of dynamical gravitational collapse. Quantum gravity may take over in these regimes to resolve the classical spacetime singularity. The quantum effects from a visible extreme gravity region could then propagate to external observers, providing a useful laboratory for quantum gravity, and implying interesting consequences for ultra-high energy astrophysical phenomena in the universe.

This volume will be of interest to graduate students and academic researchers in gravitation physics and fundamental physics, as well as in astrophysics and cosmology. It includes a review of recent research into gravitational collapse, and several examples of collapse models are worked out in detail.

Pankaj S. Joshi conducts research at the Tata Institute of Fundamental Research, Mumbai. His research interests include gravitation physics, spacetime structure and quantum gravity, and cosmology and relativistic astrophysics. He has published many research papers and books in these areas, and has held visiting faculty positions in several countries, lecturing and doing research on these topics.

Professor Joshi has an excellent international reputation for his work in the field of gravitation theory. His extensive analysis of general relativistic gravitational collapse has been widely recognized as providing significant insights into the final end states of a continual collapse, formation of visible singularities, and nature of cosmic censorship and blackholes.

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To my parents,
Arunadevi Shantilal Joshi
and
Shantilal Ramshankar Joshi

Contents

Preface

The physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star, when it collapses under its own gravity at the end of its life cycle, is one of the most important questions in gravitation theory and relativistic astrophysics today. The applications and basic theory of blackholes vigorously developed over the past decades crucially depend on this outcome.

A sufficiently massive star many times the size of the Sun would undergo a continual gravitational collapse on exhausting its nuclear fuel, without achieving an equilibrium state such as a neutron star or white dwarf. The singularity theorems in general relativity then predict that the collapse gives rise to a spacetime singularity, either hidden within an event horizon of gravity or visible to the external universe. The densities and spacetime curvatures get arbitrarily high and diverge at these ultra-strong gravity regions. Their visibility to outside observers is determined by the causal structure within the dynamically developing collapsing cloud, as governed by the Einstein field equations. When the internal dynamics of the collapse delays the horizon formation, these become visible, and may communicate physical effects to the external universe. These issues are investigated here, and the treatment is aimed at showing how such visible ultra-dense regions arise naturally and generically as the outcome of a dynamical gravitational collapse in Einstein gravity. While it predicts the existence of visible singularities; classical general relativity may no longer hold in these very late stages of the collapse, and quantum gravity may take over to resolve the classical spacetime singularity. The quantum effects from a visible, the extreme gravity region could then propagate to outside observers to provide a useful laboratory for quantum gravity. Blackholes need not form in such a scenario and there may be interesting consequences for ultra-high energy astrophysical phenomena in the universe.

The general theory of relativity, which has strong experimental support, is used here, and its basics and useful features of spacetimes are reviewed. The necessary tools are developed as needed, but a prior familiarity with general relativity would help. It is a pleasure to thank many friends and colleagues

for numerous discussions and work as cited, on the themes described here. Special thanks are due to R. Goswami and I. H. Dwivedi for their ideas and help and for our studies together. A. Mahajan and S. Khedekar helped with the manuscript.

1
Introduction

Gravitation theory and relativistic astrophysics have gone through extensive developments in recent decades, following the discovery of quasars in the 1960s, and other very high energy phenomena in the universe such as gamma ray bursts. Compact objects such as neutron stars and pulsars also display intriguing physical properties, where the effects of strong gravity fields are seen to play a fundamental role. When the masses and energy densities involved in the physical phenomena are sufficiently high, as is the case in the situations above, it has become increasingly clear that the strong gravitational fields, as governed by the general theory of relativity, play an important and much more dominant role. This gravitational dynamics must be taken into account for any meaningful description of these observed ultra-high energy objects.

A similar situation involving very strong gravitational fields, and which may be connected to some of the above phenomena, is that of a massive star undergoing a continual gravitational collapse at the end of its life cycle. This happens when the star has exhausted its nuclear fuel that provided a balance against the internal pull of gravity. This phenomenon, dominated essentially by the force of gravity, is fundamental to basic theory and astrophysical applications in blackhole physics that have received increasing attention in past decades, and also in cosmology. In the past two decades, there have been extensive investigations of gravitational collapse models within the framework of Einstein's theory of gravity, and these have provided useful insights into the final fate of a massive star.

This book is about the phenomena of gravitational collapse. Such a collapse of massive matter clouds is at the heart of the physics and astrophysics of happenings, some of which are mentioned above, where extremely high mass and energy densities are involved. For example, several models to explain gamma ray bursts are in terms of a collapsar, where the gravitational collapse of a single massive star is invoked to understand such a burst of ultra-high energy. Apart from blackhole physics, gravitational collapse is the key physical process that is fundamental to the formation of a star itself from interstellar clouds or nebulae, in the formation of galaxies and clusters of galaxies, and in structure formation in the universe as a whole. In general, gravitational collapse of a massive matter cloud would play an important role in the physical processes and a variety of happenings on a cosmic scale that involve the force of gravity in an important manner.

A continual gravitational collapse for a massive star would be the situation when the entire matter cloud collapses and shrinks under the force of its own gravity. Therefore, gravity overtakes and dominates the other three fundamental forces of nature, in particular the weak and strong nuclear forces, which generically provide the outward pressure in a star to balance it against the inward pull of gravity of the cloud, in addition to the usual thermal pressures. For massive stars, typically such a collapse takes place when the star has exhausted its nuclear fuel, and when there is no supporting force left against the force of its own gravity, which is ever present.

The final outcome of such a collapse depends on the initial mass of the star. A star with a mass lower than about two to three solar masses will stabilize as a white dwarf or neutron star after losing some of its original mass. In these cases, after an initial collapse of the cloud when the star has exhausted its nuclear fuel, the star again stabilizes at a much smaller radius due to internal balancing forces provided by either electron or neutron degeneracy pressures. For heavier stars that are several solar masses, they may again settle to a neutron star final state if the star could throw away the excess mass in the process of its evolution. However, for more massive stars, none of the above internal pressures can achieve the required balance, and a continual gravitational collapse becomes inevitable. The collapse then must proceed towards creating a spacetime singularity, as predicted by the singularity theorems of general relativity theory, which may be hidden within a blackhole or which may be visible to external observers. A spacetime singularity is a region where the physical parameters such as mass, energy densities, and the spacetime curvatures go to their extreme values and blow up, so that the usual laws of physics break down at such a singularity.

In such extreme regions, however, where the length and time scales are comparable to the Planck length and time, quantum effects become important. These must necessarily be taken into account and combined with the effects of gravity. At present, we have no mechanism or complete theory to deal with such quantum effects and the intense force of gravity together in a unified manner, namely a quantum gravity theory. However, it is widely believed that a quantum gravity theory, dealing with all forces of nature in a unified way, would take over from purely classical general relativity when the collapse reaches extreme matter densities and spacetime curvatures in its very advanced later stages. In these stages of collapse, it is very likely that when the quantum effects are incorporated together with the gravitational force, the classical spacetime singularity may be resolved, and may no longer exist in the full theory.

Gravitational collapse is thus a key phenomena for many astrophysical processes for stars or other larger systems in the universe. In particular, the very advanced stages of collapse of a massive star are occurrences in nature where the effects of both gravity and the quantum would be combined. Even if the final spacetime singularity, as predicted by classical general relativity, may be resolved, possibly through quantum gravity effects, such a collapse will necessarily give rise to spacetime regions of ultra-high mass densities and curvatures, where the physical effects will be extreme.

The important physical issue would then be whether such extreme gravity regions formed in the gravitational collapse of a massive star are visible to external observers in the universe. An affirmative answer here would mean that the physical phenomena of the gravitational collapse of a massive star could provide a very good laboratory to study quantum gravity effects in the cosmos, and this may help towards generating clues for an, as yet, unknown theory of quantum gravity. A laboratory similar to that provided by the early universe is then created in the later stages of the continual collapse of a massive star. An additional feature would be that, whereas the early universe was a unique event that happened only once, the collapse phenomena would continue to occur whenever a sufficiently massive star in the universe died on exhausting its nuclear fuel. If such ultra-strong gravity regions become visible to external observers in the spacetime, an opportunity to observe the quantum gravity effects in the universe is provided.

The answer to this is determined by the causal structure of spacetime in the vicinity of a spacetime singularity. This is actually decided by the dynamics of the gravitational collapse of the matter cloud, as it evolves from a regular initial data, defined on an initial surface, from which the collapse develops. This dynamical evolution is governed by the Einstein equations. In other words, it is only the study of the collapse dynamics of the matter clouds that would decide the visibility or otherwise of the ultra-strong gravity regions. If, as the collapse evolves, the event horizons of gravity develop much before the spacetime singularity forms, then these extreme gravity regions are hidden away from the external universe, and a blackhole forms as the collapse outcome. On the other hand, if such horizons are delayed or fail to develop during collapse, as governed by the internal dynamics of the collapsing cloud, then the scenario where the extreme gravity regions are visible to external observers occurs, and a visible naked singularity forms.

The importance of gravitational collapse processes in relativistic astrophysics was realized when Datt (1938) and Oppenheimer and Snyder (1939) used general relativity to study the dynamical collapse of a homogeneous spherical dust cloud under its own gravity. This model gave rise to the concept of a blackhole. The term *blackhole* itself was popularized only later in the 1960s. The above work established that, under idealized conditions, a collapsing cloud of matter with zero pressure will necessarily give rise to a blackhole. Such a blackhole is a region of spacetime from which no light or matter can escape away to faraway external observers, and which necessarily covers the spacetime singularity or the regions of extreme physical conditions from the external universe. Specifically, in order to create a blackhole as the final state of gravitational collapse of the star, an event horizon must develop in the spacetime earlier than the time when the final spacetime singularity forms. Such an event horizon is a one-way membrane such that light or matter can fall into the region covered by it, but cannot escape away. If the event horizon developed prior to the formation of the singularity, neither the singularity nor the collapsing matter that has fallen within it would be observable to an external observer, and a blackhole is said to have formed as the final endstate of the collapsing star. All the matter of the star is then supposed to be crushed into the infinite density singularity at the center of the blackhole.

How early and when the horizon will actually develop in a realistic collapse is determined by the dynamics of the collapsing matter, the physical conditions within the star, and the dynamical evolution of the cloud as governed by the Einstein equations of gravity. Investigations in high energy astrophysics have already used the concept of a blackhole quite extensively. However, the actual understanding of the phenomena of gravitational collapse, and the conditions under which it can lead to the blackhole formation, or otherwise, within the framework of general relativity has progressed only relatively recently.

Further to the early studies mentioned above, it was generally assumed that the final endstate of collapse of a massive star will be a blackhole only. However, several important questions remained unanswered. For example, what would be the effects of non-zero pressures, which would be certainly important in the later stages of collapse, towards determining the collapse endstate, or, how will an inhomogeneous cloud collapse, say with a physically realistic density profile that is higher at the center and decreases slowly as one moves away from the center of the star? Early work on gravitational collapse focused only on simple models with idealized conditions, assuming a totally homogeneous density within the star, zero pressures, and so on, which would not be physically realistic. For example, a realistic star must have non-zero internal pressures, and its density would be typically higher at the center, as compared with its outer layers.

These physical issues and important questions have been crucial to the foundations of blackhole physics. But, not much attention could be paid to them, mainly due to the complexity of the equations of general relativity. This is because, in general, the Einstein equations are non-linear, second order partial differential equations that are quite difficult to solve. Therefore, the only model available until the late 1960s for the dynamical gravitational collapse of a massive matter cloud was that of a homogeneous, pressureless spherical cloud. In addition, not much attention was paid to these issues by the general relativists of the 1940s and 50s, who, by and large, did not consider such ultra-high energy phenomena to be physically realistic or of much astrophysical significance.

As indicated above, it was only the discovery in the 1960s of very high energy astrophysical phenomena that generated a keen theoretical interest in the continual gravitational collapse processes. However, mathematical difficulties and the complexity of gravity theory did not allow much progress. Then, the cosmic censorship hypothesis was introduced by Penrose (1969), which conjectured that the outcome of any generic gravitational collapse of a massive star must lead necessarily only to a blackhole formation as the collapse final state. This hypothesis thus suggested that the extreme and ultra-strong gravity regions, or the spacetime singularity, must always necessarily be covered within an event horizon of gravity, and that the external observers should never be able to see the singularity. This assumption means that whatever the physical conditions and forces within the massive stars may be (for example, they may be inhomogeneous in their density distribution, the pressures may be non-zero, or they may not be totally spherical and so on), the outcome of their continual collapse must give rise to a blackhole only. In other words, this amounts to an assumption of the nature of the allowed dynamical evolutions of the collapsing clouds, namely that the Einstein equations must permit only those evolutions that create the event horizon necessarily much prior to the formation of the final singularity or the ultra-strong gravity regions. Then, the singularity would be necessarily hidden within the horizon, which is a one-way surface, not allowing it to be seen by any external observers.

The cosmic censorship conjecture thus implies that no ultra-strong gravity regions forming in continual collapse will be visible to outside observers. That is, no naked singularity will develop in the collapse, and the event horizon developing in the dynamical collapse will always manage to cover these. Hence, the outcome of any gravitational collapse is necessarily a blackhole, and external observers can never see any ultra-strong gravity regions forming in the collapse, as indicated in Fig. 1.1.

As yet, a specific mathematical formulation for cosmic censorship that has been properly defined does not exist. Then, a proof of the same would have to be obtained within the framework of Einstein's gravity theory. The cosmic censorship assumption nevertheless provided a major impetus to developments in blackhole physics, and two parallel streams of developments took place. On one hand, the theoretical properties of blackholes were developed extensively, using cosmic censorship as the basic assumption, thus

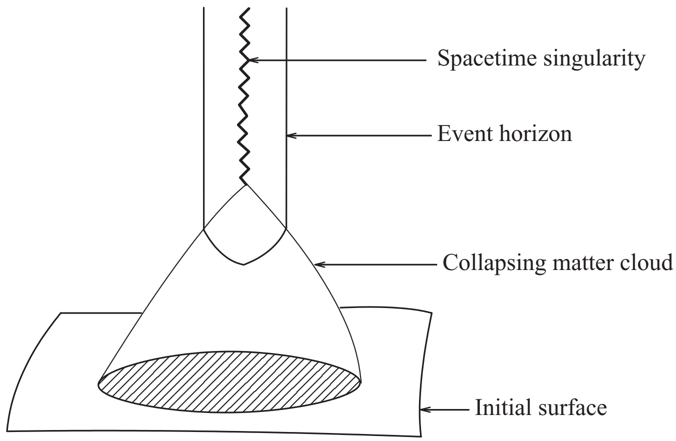

Fig. 1.1 The final outcome of a generic gravitational collapse must be a blackhole according to the cosmic censorship conjecture. Then, the eventual spacetime singularity of the collapse has to be preceded by the event horizon of gravity.

creating the laws of blackhole thermodynamics and related aspects (see for example, Hawking and Ellis, 1973). On the other hand, efforts to establish the censorship hypothesis continued, as it was clear all along that this assumption was absolutely fundamental to the theory and applications in blackhole physics, and so it needed a rigorous formulation and proof within the framework of general relativity. It is widely recognized that a proof of the censorship conjecture would place blackhole physics and its applications on a sound footing, whereas its failure would actually throw blackhole dynamics and related applications into serious doubt. Hence, the validity, or otherwise, of the cosmic censorship conjecture has remained an issue of crucial importance for all these years. The efforts to prove it have not succeeded for the past three decades, and there are even serious difficulties in formulating any rigorous mathematical version or a statement for this conjecture.

The theme that the only way out of this impasse is to study rigorously the dynamical gravitational collapse phenomena within the framework of Einstein's theory of gravity is proposed and developed here. This has been investigated extensively in the last couple of decades, and some of the issues that have been addressed include: what is the outcome of a continual gravitational collapse under physically realistic conditions, as governed by the Einstein equations? Will it be necessarily a blackhole as hypothesized by the censorship conjecture, or would it give rise to a naked singularity, where ultra-strong gravity regions forming in collapse are visible to external observers? In the latter case, would it be possible to observe the quantum gravity effects taking place in these visible ultra-strong gravity regions? Some of these issues are discussed here.

A detailed study of the collapse phenomena may be the only way towards any possible physically realistic formulation of censorship, if one exists. Such a study and investigation of collapse could also lead to novel physical insights and possibilities emerging out of the intricacies of the gravitational force. It would appear that beyond the studies so far, mainly of static and stationary solutions modeling blackholes, investigating dynamical evolutions as permitted by the Einstein equations would offer new insights into the nature of gravity. This is an arena that has been explored less, and which needs to be investigated carefully in detail.

To this end, gravitational collapse scenarios with non-zero pressures and more realistic equations of state for classes of general matter fields are considered here. A general formalism is developed to treat the spherical collapse from regular initial data. These considerations also point to why it has not been possible so far to make any definite progress on the censorship conjecture. It is seen that it is first necessary to acquire a deeper and more extensive understanding of the dynamical evolutions and gravitational collapse processes in general relativity. Recent work on studying and understanding the final fate of dynamical gravitational collapse in gravitation theory is discussed. General matter fields are considered so as to include important physical features in the collapse, such as inhomogeneities in matter distribution, non-vanishing pressures, different forms for the equations of state of the collapsing matter, and other such aspects. It is seen that in spherical gravitational collapse, given the matter initial data on an initial surface from which the collapse develops, there are the rest of the free initial data such as the velocities of the collapsing shells, and the classes of the dynamical evolutions as permitted by the Einstein equations, which lead to the final state that is either a blackhole, or a naked singularity that is a visible ultra-strong density and curvature region forming in the collapse not covered by an event horizon. The nature of the outcome depends on the regular initial data from which the collapse evolves, and the allowed dynamical evolutions in the spacetime, as permitted by the Einstein equations.

After the basics of the structure and properties of spacetimes and the essentials of relativity theory are summarized in Chapter 2, the above issues are discussed in Chapter 3. Collapsing dust clouds, which generalize and include as a special case the Oppenheimer–Snyder dust collapse models, and which give an idea of the possible outcomes of gravitational collapse in terms of a blackhole or a naked singularity, are also discussed in Chapter 3. The Oppenheimer–Snyder dust collapse scenario is included here as a special case when the cloud is homogeneous. It is seen, however, that a more realistic density profile with a density higher at the center and decreasing as one moves away from center, gives rise to a naked singularity as the collapse endstate

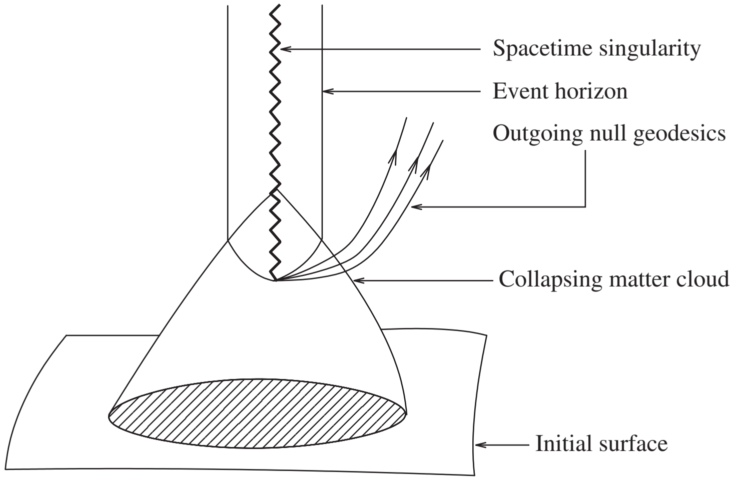

Fig. 1.2 If the collapsing cloud is inhomogeneous, with a density higher at the center, the trapped surface formation and event horizon in the collapse are delayed to give rise to a naked singularity, where the ultra-strong gravity regions are visible to outside observers.

(see Fig. 1.2). In general, it is seen that the collapse outcome depends on the nature of the initial matter profiles and the evolutions allowed by the Einstein equations. The structure of this spacetime in the homogeneous density case gives rise to the basic notion and concept of a blackhole. The dust collapse picture provides a concrete background to the possible final states of a continual gravitational collapse.

Chapter 4 then studies several useful aspects of spacetime structure, singularities and collapse, as related to the cosmic censorship hypothesis possibilities and the structure of naked singularities developing in gravitational collapse. It is pointed out that while the cosmic censorship does not hold in general relativity in the obvious sense of ruling out naked singularities from all physically realistic gravitational collapse models, any definite formulation of this hypothesis will depend on a detailed analysis of stability and genericity aspects related to collapse scenarios, and the naked singularities and blackhole phases developing as final outcomes of the gravitational collapse. Several possibilities towards any plausible formulation are discussed.

In light of the results available so far and the emerging scenario, the key physical issue is the possible final state of a massive star. The basic problem to be addressed is: what will the final outcome of the gravitational collapse of a massive star be when it collapses freely at the end of its life cycle on exhausting its nuclear fuel under the force of its own gravity? Under realistic astrophysical conditions, will it turn into a blackhole, or does it terminate as a naked singularity? Are there any observable consequences in the latter case?

These physical questions underlie many considerations here on gravitational collapse.

While theoretical properties of blackholes have been studied rather extensively, the naked singularity solutions in general relativity, arising out of dynamical collapse studies are relatively less understood as yet. It is sometimes asked how a naked singularity could arise in the collapse, allowing the light to escape from the extreme gravity regions even when the gravity fields are so strong. Some of these aspects are discussed in Chapter 5. Also explored and pointed out here are the physical features, such as the role of inhomogeneities and spacetime shear, that lead to a naked singularity rather than a blackhole as the collapse endstate. The physics that possibly causes a naked singularity in the collapse, rather than a blackhole, is examined. While it may be stated that a good understanding now exists on spherical collapse in general for a generic matter field, non-spherical collapse remains major uncharted territory. This is also closely related to the stability and genericity aspects of collapse outcomes, and these issues are discussed here.

The information loss paradox and related issues have highlighted some of the important problems with the blackhole paradigm, which also include the existence of an infinite density spacetime singularity at the center of a blackhole, leading to an instability even at the classical level, and uncertainties of the correctness, or otherwise, of the cosmic censorship conjecture. Under such a situation, a possibility worth considering could be the avoidance or delay of trapped surfaces formation as the star evolves, collapsing under gravity. This is the case when a collapse evolution to a naked singularity takes place, where the trapped surfaces do not form early enough or are avoided in the spacetime. In that case, in the late stages of the collapse, the star could radiate away most of its mass. This could then offer a way out of the blackhole conundrums, whilst also resolving the singularity problem.

As such, the outcomes of a continual collapse, namely the blackhole and naked singularity, are very different from each other in nature. The naked singularity, which is more like an event than an object in many cases, could have quite different physical properties compared with a blackhole. Therefore, the implications of the visibility of the ultra-high density and curvature regions to a faraway observer in the spacetime need to be investigated. Such a scenario offers an intriguing possibility that the quantum gravity effects may become observable during the later final stages of the collapse. This is because the ultra-strong gravity regions where quantum gravity effects take place are now no longer hidden under the event horizon, but are visible and can, in principle, communicate with external observers. This may offer interesting connections and pointers towards observational effects of quantum gravity arising from gravitational collapse. These possibilities are discussed in Chapter 5, where some implications of loop quantum gravity formalism from such a perspective are indicated.

2
The spacetime manifold

Here, the essential fundamentals of general relativity and related mathematical aspects are described. For further details, see texts such as Weinberg (1972), Misner, Thorne, and Wheeler (1973), and Wald (1984). Other necessary techniques are developed in later chapters as necessary. While defining vectors, tensors, and other quantities, we use both a local and a coordinate free global approach, and indicate how to make a transition from one to the other representation, which is useful in several situations.

In Section 2.1 the manifold model for spacetime is introduced. Basic definitions of a differentiable manifold, and various topological and orientability properties are discussed. The metric tensor and related aspects are considered in Section 2.2, and the connection on a spacetime is considered in Section 2.3. Timelike and null geodesics play a basic role in the considerations here on gravitational collapse. These are a special set of non-spacelike trajectories that represent the motion of freely falling material particles and light rays, and they clarify many properties of a spacetime. These are discussed in Section 2.4. The spacetime curvature is considered in Section 2.5, and the Einstein equations governing the dynamics of matter in the spacetime are discussed in Section 2.6. Many exact solutions have been found to the Einstein equations so far; however, the Schwarzschild and Vaidya geometries are particularly relevant to gravitational collapse scenarios, and Section 2.7 discusses these.

2.1 The manifold model

The universe is modeled as a four-dimensional spacetime M in general relativity, together with an indefinite Lorentzian metric tensor g, which has the signature $(-,+,+,+)$. Conditions ensuring physical reasonability to the spacetime model are generally assumed. These include the space and time orientability, and necessary topological regularity conditions such as the Hausdorffness and connectedness. Here, this basic model of the space-time universe that underlies Einstein's theory of gravitation is specified. The manifold model for the universe naturally incorporates the observed continuity of space and time at the classical level, and the basic principle of general relativity where the locally flat regions combine to produce a globally curved continuum. This implies that a smooth change of coordinates is possible when a transition is made from one coordinate patch to another.

2.1.1 Differentiable manifolds

The *n-dimensional Euclidian space* R^n is a collection of all n-tuples $(x^1, \ldots, x^n)$ such that $-\infty < x^i < \infty$, $i = 1, \ldots, n$, and which has the natural Euclidian metric. An *open ball* of radius r around any point x in R^n is the set of all points y such that $\mid x - y \mid < r$, where the modulus denotes the positive definite distance as defined by the Euclidian metric on R^n. The *open sets* in R^n are sets which can be expressed as a union of such open balls.

Basically, an n-dimensional differentiable manifold is a set that is locally similar to an open set of R^n. Therefore, locally Euclidian patches are glued together smoothly to obtain a space which need not be Euclidian globally.

An *n-dimensional, C^∞, real differentiable manifold* is a set M, together with a collection $\{u_\alpha, \phi_\alpha\}$, called an *atlas* for M. Here, the u_α values are subsets of M and the ϕ_α values are one–one maps of a given u_α onto an open subset in R^n, which satisfy the following.

(1) The sets u_α form a cover for M, that is, any given p in M must be in a u_α for some value of α, and

$$M = \bigcup_\alpha u_\alpha. \tag{2.1}$$

(2) Whenever two neighborhoods u_α and u_β intersect, that is, $u_\alpha \cap u_\beta \neq \phi$, then the map $\phi_\alpha \circ \phi_\beta^{-1}$ from R^n to R^n, which takes points of $\phi_\beta(u_\alpha \cap u_\beta)$ to points of $\phi_\alpha(u_\alpha \cap u_\beta)$, is infinitely differentiable in a continuous manner (a smooth C^∞-function) as a mapping between two open subsets of R^n (see Fig. 2.1).

Alternatively, it is possible to consider the map $\phi_\beta \circ \phi_\alpha^{-1}$, and the same condition again holds. Each u_α is called a local coordinate neighborhood or a *chart* where $p \in u_\alpha$ has coordinates of $\phi_\alpha(p)$ in R^n. The condition (2) above ensures that whenever an event $p \in M$ undergoes a coordinate

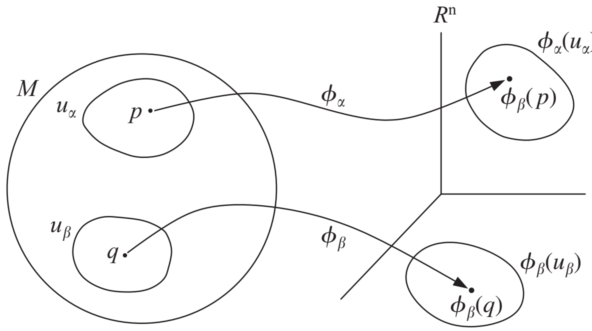

Fig. 2.1 All events p and q in the manifold have neighborhoods which are homeomorphic to subsets in R^n. The points $p, q \in M$ have coordinates of $\phi_\alpha(p)$ and $\phi_\beta(q)$. Whenever the neighborhoods in M intersect, there should be a smooth change of coordinates.

change, the change is necessarily smooth. That is, if $\{x^i\}$ and $\{y^i\}$ are local coordinates of $p \in M$ in u_α and u_β respectively, then the functions $x^i = x^i(y^1, \ldots, y^n)$ are C^∞-functions from R^n to R^n. A *maximal* or *complete atlas* is chosen for the spacetime manifold M, that is, if $\{u_\alpha, \phi_\alpha\}$ is an atlas for M, one selects for M the atlas that consists of all other atlases that are compatible with $\{u_\alpha, \phi_\alpha\}$. This implies that their union with $\{u_\alpha, \phi_\alpha\}$ is also a C^∞-atlas.

This implies that one has included all possible, mutually compatible coordinate systems for the manifold M. A C^r-manifold is defined in a similar way, where it is required that the transition functions $\phi_\alpha \circ \phi_\beta^{-1}$ are r-times continuously differentiable, where a continuous function is denoted by C^0.

The Euclidian plane R^2, or Euclidian space R^n, is, in itself, a manifold as it is covered by a single chart R^n, where ϕ would be the identity map with the coordinate range $-\infty < x^i < \infty$ for $i = 1, \ldots, n$. Another example of such a manifold is the two-sphere S^2 defined by

$$S^2 = \{(x^1,\ x^2,\ x^3) \in R^3 \mid (x^1)^2 + (x^2)^2 + (x^3)^2 = 1\}. \tag{2.2}$$

The six hemispherical open sets $O_i^\pm$ for $i = 1, 2, 3$ are given by $O_i^\pm = \{(x^1, x^2, x^3) \in S^2 \mid \pm x^i > 0\}$, which cover S^2. Each $O_i^\pm$ is mapped onto the open disk $\{(x, y) \in R^2 \mid x^2 + y^2 < 1\}$ by the projection maps such as $f_1^+(x^1, x^2, x^3) = (x^2, x^3)$. The overlap functions $f_i^\pm \circ (f_j^\pm)^{-1}$ are C^∞-functions in their domain of definition. Thus, S^2 is a two-dimensional, C^∞-manifold that cannot be covered by a single coordinate system. Similarly, the sphere S^n in n-dimensions is also a differentiable manifold.

2.1.2 Vectors and one-forms

A function $f : M \to R^n$ is called *differentiable* if the map $f \circ \phi_\alpha^{-1}$ is a C^∞-map for all charts ϕ_α as a map from R^n to R^n; C^r-functions can be defined similarly (Spivak, 1965).

Suppose now M and M' are two differentiable manifolds with ϕ_α and ψ_α denoting charts of M and M' respectively. A map $h : M \to M'$ is called C^r-*differentiable* if $\psi_\alpha \circ h \circ \phi_\alpha^{-1}$ is always C^r-differentiable as a map from R^n to R^n for all α. If the dimension of M is n and that of M' is n' with $n > n'$, then the map h cannot be one–one. However, if h is one–one, onto, and continuous from M to M' such that h^{-1} is also a continuous map, then h is called a *homeomorphism*. If a homeomorphism and its inverse are both C^r-maps, then it is called a C^r-*diffeomorphism*.

A C^k-*curve* in M is a C^k-map from an interval of R into M. A *vector* (or a contravariant vector) $(\partial/\partial t)_{\lambda(t_0)}$, tangent to a C^k-curve $\lambda(t)$ at a point $\lambda(t_0)$, is an operator from the space of all smooth functions on M into R:

$$\left(\frac{\partial}{\partial t}\right)_{\lambda(t_0)} (f) = \left(\frac{\partial f}{\partial t}\right)_{\lambda(t_0)} = \lim_{s \to 0} \frac{f[\lambda(t+s)] - f[\lambda(t)]}{s}, \tag{2.3}$$

where s denotes a small increment of the parameter t. This is $d(f \circ \lambda)/dt$, which is the derivative of f in the direction of $\lambda(t)$ with respect to parameter t. If $f = t$, where t is the parameter along the curve,

$$\left(\frac{\partial}{\partial t}\right)_{\lambda} (t) = 1. \tag{2.4}$$

If the x^i values are local coordinates in a neighborhood of $p = \lambda(t_0)$, then

$$\left(\frac{\partial f}{\partial t}\right)_{\lambda(t_0)} = \frac{dx^i}{dt} \frac{\partial f}{\partial x^i} |_{\lambda(t_0)}, \tag{2.5}$$

where a repeated index means summation over the values $1, \ldots, n$. (This *summation convention* is used throughout.) Therefore, every tangent vector at $p \in M$ is expressed as a linear combination of the coordinate derivatives, which are $(\partial/\partial x^1)_p, \ldots, (\partial/\partial x^n)_p$. Conversely, any linear combination of these operators that are partial derivatives with respect to coordinates can be chosen, namely, $V^i(\partial/\partial x^i)_p$, with the values of V^i being any numbers. It is then possible to find a curve which admits this linear combination as a tangent (see for example, Wald, 1984). The vectors $(\partial/\partial x^j)_p$ are linearly independent (if not, then there are numbers V^i such that

$$V^i \left(\frac{\partial}{\partial x^i}\right)_p = 0, \tag{2.6}$$

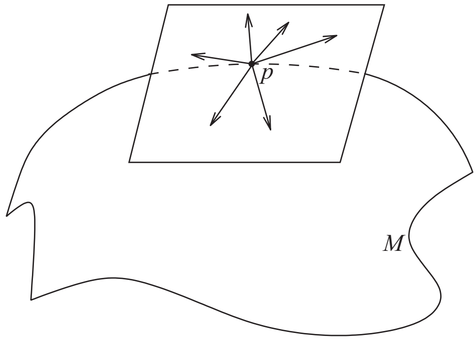

Fig. 2.2 The tangent space T_p at a point $p \in M$, which gives the set of all directions at that point.

with at least one V^i being non-zero, and applying this to the coordinate functions $x^1, \ldots, x^n$ gives $V^i = 0$ for all i, a contradiction). Therefore, the vectors $(\partial/\partial x^j)$ span the vector space T_p, the space of all tangent vectors at p (see Fig. 2.2). The vector space structure here is defined by

$$(\alpha \boldsymbol{X} + \beta \boldsymbol{Y})f = \alpha(\boldsymbol{X}f) + \beta(\boldsymbol{Y}f), \tag{2.7}$$

for $\alpha, \beta \in R$ and $\boldsymbol{X}, \boldsymbol{Y} \in T_p$, where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are vectors at p; T_p is also called the *tangent space* at p. The basis $\{(\partial/\partial x^i)_p\}$ is called a *coordinate basis* of T_p. A general basis is denoted by $\{\boldsymbol{e}_i\}$, where $i = 1, \ldots, n$, are linearly independent vectors. Then, for any vector $\boldsymbol{V} \in T_p$,

$$\boldsymbol{V} = V^i \boldsymbol{e}_i, \tag{2.8}$$

where the numbers V^i are called the components of $\boldsymbol{V}$ with respect to the basis $\boldsymbol{e}_i$. In a coordinate basis, $V^i = dx^i/dt$. Again, $\{\partial/\partial x^i\}$ forms a basis of T_p which means the dimension of T_p is n.

For the tangent space T_p at $p \in M$, the vector space of all dual vectors at p, also called *covariant vectors* or one-forms at p, can be naturally defined. A one-form $\boldsymbol{\omega}$ at p is a real-valued linear functional on T_p, denoted by $\boldsymbol{\omega}(\boldsymbol{X}) \equiv \langle \boldsymbol{\omega}, \boldsymbol{X} \rangle$, and the linearity condition implies

$$\langle \boldsymbol{\omega}, \alpha \boldsymbol{X} + \beta \boldsymbol{Y} \rangle = \alpha \langle \boldsymbol{\omega},\ \boldsymbol{X} \rangle + \beta \langle \boldsymbol{\omega}, \boldsymbol{Y} \rangle. \tag{2.9}$$

Given a tangent space basis $\{\boldsymbol{e}_a\}$, a unique set of one-forms $\{\boldsymbol{e}^a\}$ is given by the condition that the one-form $\boldsymbol{e}^b$ maps a vector $\boldsymbol{V}$ into V^b, that is, the bth component of $\boldsymbol{V}$ in the basis $\boldsymbol{e}_a$. Therefore,

$$\langle \boldsymbol{e}^b, \boldsymbol{V} \rangle = V^b, \tag{2.10}$$

where $a, b, \ldots$ and $i,\ j, \ldots$ denote indices for vectors and tensors.

From the above,

$$\langle \boldsymbol{e}^a, \boldsymbol{e}_b \rangle = \delta^a_b, \tag{2.11}$$

where the right-hand side is the Kronecker delta function. The linear combinations of one-forms $\boldsymbol{\omega}$ and $\boldsymbol{\eta}$ are defined by

$$\langle \alpha\boldsymbol{\omega} + \beta\boldsymbol{\eta}, \boldsymbol{V} \rangle = \alpha\langle \boldsymbol{\omega}, \boldsymbol{V} \rangle + \beta\langle \boldsymbol{\eta}, \boldsymbol{V} \rangle, \tag{2.12}$$

with $\alpha, \beta \in R$. Then, $\{\boldsymbol{e}^a\}$ is a basis for the space of all one-forms at p because any one-form $\boldsymbol{\omega}$ can be written as $\boldsymbol{\omega} = \boldsymbol{\omega}_a \boldsymbol{e}^a$ with

$$\omega_a = \langle \boldsymbol{\omega}, \boldsymbol{e}_a \rangle. \tag{2.13}$$

Therefore, the set of all one-forms at the event p forms a vector space at p, the *dual* of T_p, and is denoted by T^*_p. The basis $\boldsymbol{e}^a$ is a *dual basis* to $\boldsymbol{e}_a$. If $\boldsymbol{\omega} \in T^*_p$ and $\boldsymbol{V} \in T_p$, then

$$\langle \boldsymbol{\omega}, \boldsymbol{V} \rangle = \langle \omega_a \boldsymbol{e}^a, V^b \boldsymbol{e}_b \rangle = \omega_a V^b \delta^a_b = \omega_a V^a. \tag{2.14}$$

A *vector field* $\boldsymbol{V}$ on a manifold M is an assignment of a tangent vector $\boldsymbol{V}_p$ at each $p \in M$. The vector field is said to assign vectors smoothly if, for each smooth function f on M, the function $\boldsymbol{V}(f)$, the directional derivative of f along the vector $\boldsymbol{V}_p$, is also smooth on M at each point p. The coordinate basis vector fields $\partial/\partial x^i$ are smooth, and so a vector field will be smooth provided that its coordinate components V^i are smooth functions. Given two vector fields $\boldsymbol{V}$ and $\boldsymbol{W}$, a new vector field, called their *commutator* $[\boldsymbol{V}, \boldsymbol{W}]$, is defined by

$$[\boldsymbol{V}, \boldsymbol{W}](f) = \boldsymbol{V}[\boldsymbol{W}(f)] - \boldsymbol{W}[\boldsymbol{V}(f)]. \tag{2.15}$$

The commutator for any two coordinate basis vector fields vanishes. If f and g are any two smooth functions, it can be seen that $[\boldsymbol{V}, \boldsymbol{W}](f + g) = [\boldsymbol{V}, \boldsymbol{W}](f) + [\boldsymbol{V}, \boldsymbol{W}](g)$ and that $[\boldsymbol{V}, \boldsymbol{W}](\alpha f) = \alpha[\boldsymbol{V}, \boldsymbol{W}](f)$ for any $\alpha \in R$. It can be shown that

$$[\boldsymbol{V}, \boldsymbol{W}](fg) = f[\boldsymbol{V}, \boldsymbol{W}](g) + g[\boldsymbol{V}, \boldsymbol{W}](f), \tag{2.16}$$

which is the product property. By expanding in a coordinate basis it is seen that the commutator $[\boldsymbol{V}, \boldsymbol{W}]$ will be a smooth vector field if and only if both $\boldsymbol{V}$ and $\boldsymbol{W}$ are smooth. Note that $[\boldsymbol{V}, \boldsymbol{V}] = 0$ and that $[\boldsymbol{V}, \boldsymbol{W}] = -[\boldsymbol{W}, \boldsymbol{V}]$. Furthermore, the commutator is linear in each of its arguments with respect to addition, that is

$$[\boldsymbol{V}_1 + \boldsymbol{V}_2, \boldsymbol{W}] = [\boldsymbol{V}_1, \boldsymbol{W}] + [\boldsymbol{V}_2, \boldsymbol{W}]. \tag{2.17}$$

Any smooth function f on M defines a one-form df, called the *differential* of f, by the rule

$$\langle df, \boldsymbol{V} \rangle \equiv \boldsymbol{V} f. \tag{2.18}$$

Therefore, in a coordinate basis,

$$\langle df, \boldsymbol{V} \rangle = V^a \frac{\partial f}{\partial x^a}. \tag{2.19}$$

The local coordinate functions $(x^1, \ldots, x^n)$ are used to define a set of one-forms $(dx^1, \ldots, dx^n)$, which is a basis dual to the coordinate basis because

$$\left\langle dx^a, \frac{\partial}{\partial x^b} \right\rangle = \frac{\partial x^a}{\partial x^b} = \delta^a_b. \tag{2.20}$$

Also,

$$df = \left\langle df, \frac{\partial}{\partial x^a} \right\rangle dx^a = \frac{\partial f}{\partial x^a} dx^a, \tag{2.21}$$

which is the usual definition of the differential df.

If f is a non-constant function, the surfaces $f = \text{const.}$ define an $(n-1)$-dimensional submanifold of M. Consider the set of all the vectors $\boldsymbol{V} \in T_p$ such that

$$\langle df, \boldsymbol{V} \rangle = \boldsymbol{V} f = 0, \tag{2.22}$$

then the vectors $\boldsymbol{V}$ are tangent to curves in the $f = \text{const.}$ submanifold, through p. Therefore, the differential df is normal to the surface $f = \text{const.}$ at p.

2.1.3 Topological structure

A C^∞-maximal atlas on a spacetime manifold M induces a natural *topology on* M, given by the companion Euclidian space by requiring each ϕ_α to be a homeomorphism. Therefore, the open sets in M are pre-images of open sets in R^n and their unions. Then, the collection $\{u_\alpha\}$ provides a basis for the spacetime topology, where R^n has its canonical topology, defined by the metric

$$d(x, y) = \left[(x_1 - y_1)^2 + \cdots + (x_n - y_n)^2\right]^{1/2} \tag{2.23}$$

for any $x, y \in R^n$.

Several topological regularity conditions that are assumed for a physically reasonable spacetime manifold are now given. The spacetime is assumed to be *Hausdorff*, that is, given p and q with $p \neq q$ in M, there are disjoint open sets u_α and u_β in M such that $p \in u_\alpha$ and $q \in u_\beta$. Physically interesting spacetime examples such as the Schwarzschild geometry and Robertson–Walker models are Hausdorff. This is a reasonable requirement on a spacetime which ensures the uniqueness of limits of convergent sequences, and incorporates the intuitive notion of distinct spacetime events.

Next, the spacetime M has *no boundary*. A boundary represents, in a sense, the 'edge' of the universe, which is not detected by any astronomical observations. Mathematically, it is common to have manifolds without a boundary. For example, for a two-sphere S^2 in R^3, no point in S^2 is a boundary point in the induced topology as implied by the natural topology on R^3, because all neighborhoods of any $p \in S^2$ are contained within S^2 in this induced topology. Assume M to be *connected*, that is, $M = X_1 \cup X_2$, with X_1 and X_2 being two open sets and $X_1 \cap X_2 = \phi$ is not possible. This is because disconnected components of the universe cannot interact by means of any signals, and the observations are confined to the connected component where the observer is situated. But, M could be either simply connected or multiply connected. For further discussion on multiply connected spacetimes and the notion of a wormhole in the Schwarzschild geometry, see Wheeler (1962, 1964) and Misner, Thorne, and Wheeler (1973). Such wormholes are like 'handles' in the multiply connected topology of space and can connect widely separated regions in space.

It is known, however, that such wormholes are not stable and collapse as soon as created, unless the violation of the energy condition in an averaged sense is allowed, thus implying negative energy fields (see for example, Deutsch and Candelas, 1980; Lee, 1983; Morris, Thorne, and Yurtsever, 1988). Therefore, a wormhole may be stabilized only by shifting the energy of vacuum to be negative by some quantum processes. The process of topology change could also give rise to a multiply connected spacetime. It is not clear if the topology of space could change while it evolves in time, and if so, what physical agencies cause it. A topology change can affect the structure of spacetime severely to cause naked singularities (Joshi and Saraykar, 1987).

A spacetime is assumed to be *non-compact*, because compact spacetimes violate causality and admit closed timelike curves. One could then enter one's own past, which is considered to be highly unphysical. Usually, M is also taken to be paracompact. An atlas $\{u_\alpha, \phi_\alpha\}$ is called *locally finite* if there is an open set containing every $p \in M$ that intersects only a finite number of the sets u_α. A manifold M is called *paracompact* if, for every atlas $\{u_\alpha, \phi_\alpha\}$, there is a locally finite atlas $\{O_\beta, \psi_\beta\}$ with each O_β contained in some u_α. For a further discussion on these topological concepts, see Simmons (1963) and Willard (1970).

For a connected, Hausdorff manifold, the paracompactness property is equivalent to the existence of a countable base for the topology of M. The existence of a Lorentz metric globally on M implies any Hausdorff manifold with a C^r Lorentz metric tensor must be paracompact (Geroch, 1968b).