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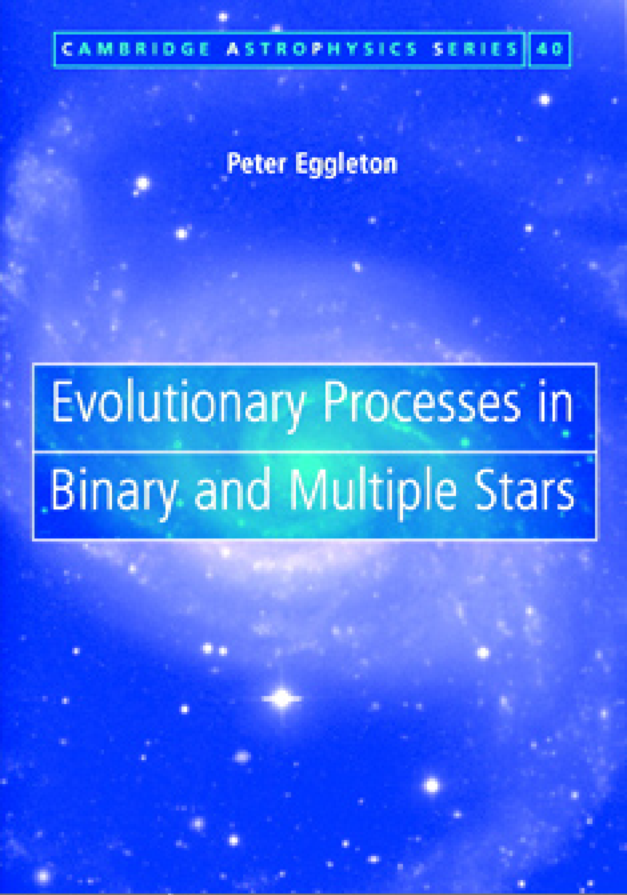
Peter Eggleton

Evolutionary Processes in Binary and Multiple Stars



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Binary systems of stars are as common as single stars. Stars evolve primarily by nuclear reactions in their interiors, but a star with a binary companion can also have its evolution influenced by the companion. Multiple star systems can exist stably for millions of years, but can ultimately become unstable as one star grows in radius until it engulfs another.

This volume discusses the statistics of binary stars; the evolution of single stars; and several of the most important kinds of interaction between two (and even three or more) stars. Some of the interactions discussed are Roche-lobe overflow, tidal friction, gravitational radiation, magnetic activity driven by rapid rotation, stellar winds, magnetic braking and the influence of a distant third body on a close binary orbit. A series of mathematical appendices gives a concise but full account of the mathematics of these processes.

PETER EGGLETON is a physicist at the Lawrence Livermore National Laboratory in California. Following his education in Edinburgh, he obtained his Ph.D. in Astrophysics from the University of Cambridge in 1965. He lectured for a short period at York University before returning to the University of Cambridge to conduct research from 1967 to 2000 as a Fellow of Corpus Christi College. In 2000, he took up his current position at LLNL. He is well known throughout the community as one of the most knowledgeable experts in binary star evolution.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521855570

First published in print format 2006

ISBN-13 978-0-511-22308-2 eBook (Adobe Reader)
ISBN-10 0-511-22308-0 eBook (Adobe Reader)

ISBN-13 978-0-521-85557-0 hardback
ISBN-10 0-521-85557-8 hardback

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Preface

This book is intended for those people, perhaps final-year undergraduates and research students, who are already familiar with the terminology of stellar astrophysics (spectral types, magnitudes, etc.) and would like to explore the fascinating world of binary stars. I hope it will also be useful to those whose main astrophysical interests are in planets, galaxies or cosmology, but who wish to inform themselves about some of the basic blocks on which much astronomical knowledge is built. I have endeavoured to put into one book a number of concepts and derivations that are to be found scattered widely in the literature; I have also included a chapter on the internal evolution of *single* stars.

In the interest of keeping this volume short, I have been brief, some might say cursory, in surveying the enormous literature on observed binary stars. It is almost a truism that theoretical ideas stand or fall by comparison with observation. My intention is to produce a second volume, with my colleagues Dr Ludmila Kiseleva-Eggleton and Dr Zhanwen Han, in which individual binary and triple stars that rate less than a line in this volume will be discussed in the paragraph or two each, at least, which they deserve. In addition, the synthesis of large theoretical populations of binary stars will be discussed. Some individual binaries can be seen as flying entirely in the face of the theoretical ideas outlined here – see OW Gem, Section 2.3.5. If I took at face value the notion that one well-measured counter-example is all that is needed to demolish a theory, then I would have given up long ago. Rather, I think, it is necessary to persevere: not be paralysed by disagreement with observation, but also not to sweep disagreement under the carpet.

A number of problems that have to be considered may well be capable of being answered only by detailed numerical modelling, constructing three-dimensional models of a whole star, or of a pair of stars in a binary. Massive computer resources will be needed for such investigations; for that reason I moved from Cambridge University to the Lawrence Livermore National Laboratory, California, where such resources are being developed. This Laboratory has started the 'Djehuty Project' – named after the Egyptian god of astronomy – to pursue this long-term goal. We hope that this project will supplement, though it cannot entirely replace, the simple ideas which this book discusses.

I am very grateful to many colleagues who have been generous of their time in discussing the issues of binary-star evolution. Drs Zhanwen Han, Onno Pols, Klaus-Peter Schröder, Chris Tout and Ludmila Kiseleva-Eggleton have kindly supplied some figures, as well as much insight. I particularly wish to thank Prof. Piet Hut for his careful and critical reading of the manuscript, and suggestions for improvement, and Drs Kem Cook and Dave Dearborn for their patience in allowing me to pursue this topic.

This work was performed under the auspices of the US Department of Energy, National Nuclear Security Administration by the University of California, Lawrence Livermore National Laboratory under contract No W-7405-Eng-48; and much use was made of the archive at the Centre de Données astronomiques de Strasbourg.

1

Introduction

1.1 Background

Because gravity is a long-range force, it is difficult to define precisely the concept of an 'isolated star' – and consequently also the concept of a binary or triple star. Nevertheless, many stars are found whose closest neighbouring star is a hundred, a thousand or even a million times closer than the average separation among stars in the general neighbourhood. Such pairings of stars are expected to be very long lived. There also exist occasional local clusterings of perhaps a thousand to a million stars, occupying a volume of space which would much more typically contain only a handful of stars. These clusters can also be expected to be long lived – although not as long lived as an 'isolated' binary, since the combined motion of stars in a large cluster causes a slow evaporation of the less massive members of the cluster, which gain kinetic energy on average from close gravitational encounters with the more massive members. Intermediate between binaries and clusters are to be found small multiple systems containing three to six members, and loose associations containing somewhat larger numbers. Starting from the other end, some clusters may contain sub-clusters, and perhaps sub-sub-clusters, down to the scale of binaries and triples.

Even with the naked eye, a handful of the 5000 stars visible can be seen to be double; and in the northern hemisphere two clusters of stars, the Hyades and the Pleiades, are quite recognisable. But some 2000 naked-eye stars are known to be binary (or triple, quadruple, etc.) by more detailed measurement – astrometric, spectroscopic or photometric. Observation in other wavelength ranges, such as radio, infrared, ultraviolet and X-rays, reveal further and more exotic binary companions, not so many in number, but of unusual interest. The naked-eye stars are only a tiny fraction of all the stars in our Galaxy ($\sim 10^{11}$), but are reasonably representative as far as the incidence of binarity is concerned.

Sometimes the two components are so close together as to be virtually touching; sometimes they are so far apart as to be virtually independent. Measured orbital periods range from hours (or even minutes) up to centuries. Many must have longer periods still, not yet determined but up to millions of years. The evolution of the two components of such pairs has attracted increasing interest over the last fifty years. The presence of a binary companion, if the orbital period is a few years or less, may make the evolution of a star very different from what it would have been if the star were effectively isolated. A number of these differences are now fairly well understood, but although some evolutionary problems which used to trouble astrophysicists, such as the 'Algol paradox', have been largely resolved, several still remain. New observations add new problems considerably faster than they confirm the resolution of older problems. It should be kept in mind that even single stars present many evolutionary problems, and so it is not surprising that many binary stars do.

Questions about binary stars can be divided very loosely into two categories, 'structural' and 'evolutionary'. For a particular type of binary star one can ask what physical processes are currently going on, that give this type of star its particular characteristics. In cataclysmic variables such as novae, for instance, there is little doubt that a fairly normal main sequence star of rather low mass is being slowly torn apart by the gravitational field of a very close white dwarf companion. But one can also ask how such binaries started, and subsequently evolved, so that these processes can currently take place. This evolutionary question can be harder to answer, because most evolutionary processes are very slow. An obvious further evolutionary question is: 'What will the future evolution of such systems be, up to some long-lived final state?' This book attempts to summarise progress in understanding the kind of long-term evolutionary processes involved. In the interest of brevity it will be necessary to quote, and to take for granted rather than to discuss, most of the much more substantial literature on structural problems. However, one aspect of binary stars that might be labelled 'structural', but which is certainly of vital importance for evolutionary discussions, is the determination of such fundamental parameters as masses, radii, etc.

1.2 Determination of binary parameters

If we are interested in determining the masses and radii of stars, then we have to turn almost right away to *binary* stars, since it is only by measuring orbital motion under gravity, and by measuring the shape and depth of eclipses, that we are able to determine these quantities to a good accuracy – one or two per cent in favourable cases; see Hilditch (2001). Analysis of the spectrum of an *isolated* star can determine such useful quantities as the star's surface temperature, gravity and composition. This is done by comparing the observed spectrum, preferably not just in the visible region of wavelengths but also in the ultraviolet (UV) and infrared (IR), with a grid of computed spectra for a range of temperatures, gravities and compositions. However, we do not get a mass from this process, or a radius, only the combination that gives the gravity – except in the special case of white dwarfs, where there is expected to be a tight radius–mass relation (Section 2.3.2) so that both mass and radius are functions only of gravity.

If we have an accurate parallax, as from the Hipparcos satellite, we can get closer to determining the mass of an isolated star, because the distance, the temperature (from spectral analysis), and the apparent brightness give us the radius; and hence the gravity (also from spectral fitting) gives us the mass. However, even if the parallax is good to $\sim$1%, the gravity is much less accurate, because spectra are usually nothing like so sensitive to gravity as they are to temperature. Perhaps an accuracy of $\sim$25% is achievable.

The parameters of binary systems are generally obtained from astrometric, or spectroscopic, or photometric observations, and in favourable cases by a combination of two, or even all three, of these methods. Note that terms such as 'astrometric' and 'photometric', coined originally to refer to observations in the visible portion of the electromagnetic spectrum, are now generally used to cover all parts of the spectrum, for instance radio and X-rays. If the two components of a binary are so far apart in the sky as to be resolvable from each other, which means at visual wavelengths more than $\sim 0.1''$ (0.5 μrad) apart, then the system is a 'visual binary' or 'VB', and careful astrometry, sometimes over a century or more, can reveal the orbit. Visual binaries tend to have long periods because short-period orbits are generally not resolvable. Only for systems within $\sim$5 pc of the Sun (about 50 in number) could a separation of $0.2''$ correspond to a period of $\lesssim$1 year. The upper limit of well-determined

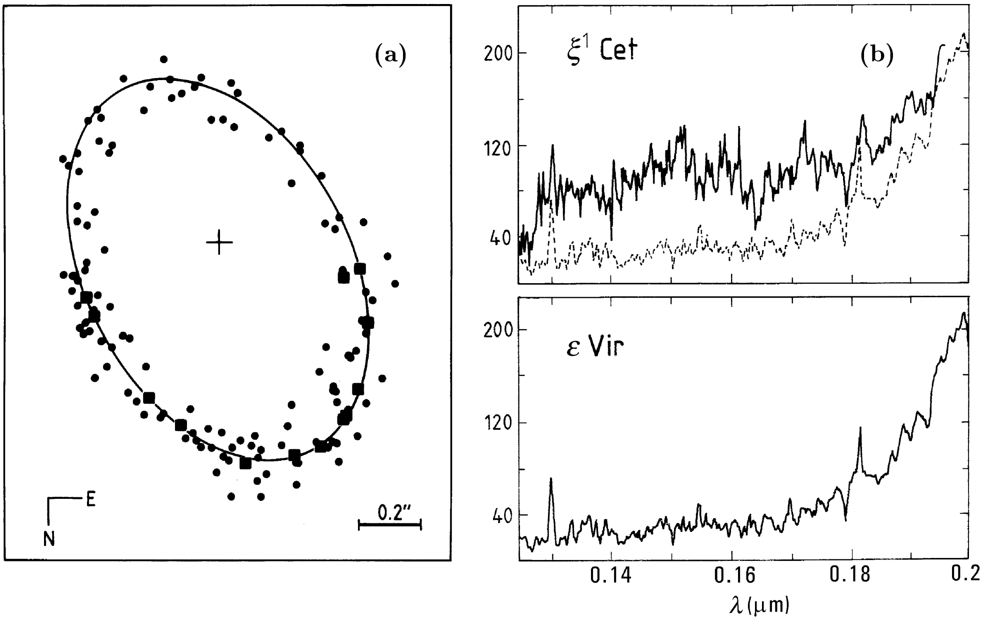

Figure 1.1 (a) The orbit of HR 3579 (F5V+G5V) from visual (dots) and speckle (square) measurements of relative position. The scatter of speckle points about the best-fit curve ($P = 21.8$ yr, $e = 0.15$, $a/D = 0.66''$, $i = 130°$) is much less than for the visual points. From Hartkopf *et al.* (1989). (b) The UV spectrum of the G8III stars ϵ Vir (bottom panel) and ξ^1 Cet (top panel, with ϵ Vir repeated). For 0.18–0.7 μm (not all shown here) the spectra are very similar. The UV excess evident in ξ^1 Cet for 0.13–0.17 μm is attributable to a white dwarf companion. From Böhm-Vitense and Johnson (1985).

visual orbital periods is about 100 years, because good accuracy is only achievable if the VB has consistently been followed for at least two full orbits. There are many orbits in the literature with periods up to 1000 years, or even longer, but these must be considered tentative – extremely tentative if the period is greater than 200 years.

Visual orbits are usually *relative* orbits, the position of one component being measured relative to the other (Fig. 1.1a). Visual orbits have been catalogued by Worley and Douglas (1984), and speckle measurements by McAlister and Hartkopf (1988). These and many other relevant catalogues can be found on the website of the Centre des Données astronomique de Strasbourg (http://cdsweb.u_strasbg.fr). From visual orbits one can determine the period (P), the eccentricity (e), the inclination (i) of the orbit to the line of sight, and the *angular* semimajor axis, i.e. the ratio of the semimajor axis a to the distance D. One can then determine M/D^3, where M is the total mass, from Kepler's law:

$$\frac{GM}{a^3} = \left(\frac{2\pi}{P}\right)^2, \quad \text{so} \quad \frac{GM}{D^3} = \left(\frac{2\pi}{P}\right)^2 \left(\frac{a}{D}\right)^3 . \tag{1.1}$$

If the VB is near enough, D may be obtainable from the parallax. For Earth-based measurements, parallaxes of less than $0.1''$ are not reliable, but this has been improved by more than an order of magnitude with space-based measurements from the Hipparcos satellite. If the orbits of both the components of a visual binary can be measured *absolutely*, i.e. each orbit relative to a background of distant and approximately 'fixed' stars, then the mass ratio of the two components can further be determined. We still do not obtain the individual masses, however, unless D is separately determinable.

Even if only one component of a binary is visible at all, an astrometric orbit may in favourable cases be found by observing that the position of a star has a cyclic oscillation superimposed on the combination of its parallactic motion and its linear proper motion relative to the 'fixed' stars, i.e. faint stars most of which do not move measurably and so can be assumed to be distant. Such astrometric binaries can yield P, e and i, but information on masses is convolved with the unknown mass ratio, and also with the parallax which may or may not be measurable even if the astrometric orbit is measurable.

Some VBs can be recognised even when neither component shows measurable orbital motion. If two stars, not necessarily *very* close together on the sky, show the same substantial linear proper motion relative to the 'fixed' background, it is likely that (a) they are physically related, and (b) fairly nearby, with measurable parallaxes. Usually these parallaxes agree, confirming the reality of the pair. Such pairs are called 'common proper motion' (CPM) pairs. The two nearest stars to the Sun, V645 Cen (Proxima Cen) and α Cen, are over 2° apart, but have the same rapid proper motion and large parallax. To be pedantic, (a) they are so near the Sun, and so far apart on the sky, that actually their proper motions and parallaxes are measurably different at the 1% level, and (b) α Cen is itself a VB of two Solar-type stars, with semimajor axis $17.5''$ and period 80 years, so that the proper motion of V645 Cen has to be compared with the proper motion of the centre of gravity (CG) of the α Cen pair. The period of the orbit of V645 Cen about the CG of the triple system can be expected to be about 1 megayear.

Common proper motion pairs are usually sufficiently wide that they might appear to be of little relevance to this book, which deals with pairs sufficiently close together that one component can influence the other's evolution. However the presence of a CPM companion can often reveal information on both components that would not be available if they were not paired. Several *close* pairs have a distant CPM companion; and if for example this companion has a character that suggests that it is fairly old, then one can reasonably conclude that the close binary is also fairly old. This may not be evident from the close binary alone, since the components in it may have interacted in ways that disguise the age of the system.

Modern techniques such as speckle interferometry (Labeyrie 1970, McAlister 1985), can resolve components with substantially smaller angular separations than conventional astrometry, and thus determine visual orbits of shorter period. The major limitation on resolving close components astrometrically is atmospheric 'seeing', the blurring effect of turbulence in the Earth's atmosphere. This distorts the image on a timescale of $\sim$0.05 s. In the speckle technique the image is recorded many times a second, and so the time variation of the point-spread function can be followed and allowed for in a Fourier deconvolution. The technique of adaptive optics (Babcock 1953, Beckers 1993) is an alternative way of eliminating seeing, by continuously adapting the shape of the mirror in response to the deformation of the image of a reference point source, either a nearby single star or the back-scattered light of a laser beam pointing along the telescope. Both techniques can give resolution down to the limit of diffraction, $\sim 0.01''$ at visual wavelengths on a modern 8 m class telescope. By combining the light from two or more separate telescopes, the technique of 'aperture synthesis', long used in radio astronomy, can nowadays be applied to optical wavelengths (Burns *et al.* 1997), and should be capable of sub-milliarcsecond resolution, so that one might hope to see directly both components of nearby short-period binaries.

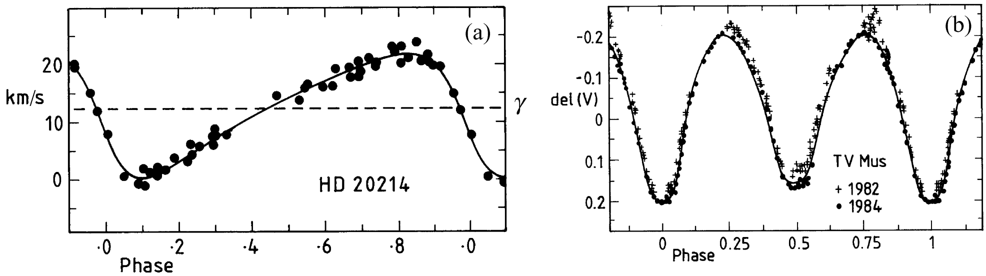

Figure 1.2 (a) The radial velocity curve of the K giant star HD20214. The rms scatter about the mean curve is only ~ 0.2 km/s. Orbital parameters are $P = 407$ days, $e = 0.41$, $f = 0.040\,M_\odot$. From Griffin (1988). (b) The light curve of a contact binary TV Mus ($P = 0.446$ days, $e = 0$, $i = 78.9°$, $R_1/a = 0.59$, $R_2/a = 0.27$, $M_1/M_2 = 7.2$, $T_1/T_2 = 0.98$). A slight variation in brightness over two years, and a small distortion in the secondary eclipse, may be due to starspots. From Hilditch *et al.* (1989).

Systems may be recognisable as spectroscopic binaries (SBs) either because the spectrum is composite (Fig. 1.1b), or because it shows radial velocity variations (Fig. 1.2a), or both. In a composite spectrum, one might see for instance a combination of the relatively broad lines of H and He characteristic of a B dwarf with the narrow lines of Fe and other metals characteristic of a G or K giant. Alternatively, a star whose spectrum at visual wavelengths may seem like a K giant may be found, at UV wavelengths, to have an excess flux that can be attributed to a hot companion, sometimes even a white dwarf (Fig. 1.1b). It is not easy to disentangle composite spectra reliably, since things other than a stellar companion (for example a corona, a circumstellar disc or a dust shell) may contribute to an excess either in the UV or the IR. Even if the spectrum seems definitely a composite of two stellar spectra, we learn only that the star is a binary; we do not obtain information about the orbit unless one spectrum at least shows a variable radial velocity, consistent with Doppler shift due to motion in a Keplerian orbit.

Orbits of 1469 SBs have been catalogued in the important compilation of Batten, Fletcher and McCarthy (1989). The number of orbits is increasing rapidly, perhaps already at a rate of one or two hundred a year, and no doubt with greater rapidity in the future, partly because of cross-correlation techniques and partly because of the much-increased sensitivity of detectors. Commonly SBs are single-lined ('SB1'), but the radial velocity variation of the single spectrum seen (as in Fig. 1.2a) allows P and e to be obtained and also the amplitude K of the radial velocity variation, or equivalently (as is usual for radio pulsars) the projected semi-major axis ($a \sin i \propto K P \sqrt{1 - e^2}$). Information on masses is contained in a single function, the mass function f, convolving both of the masses with the unknown orbital inclination i:

$$f_1 = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{K_1^3 P (1 - e^2)^{3/2}}{2\pi G} = 1.0385 \times 10^{-7} K_1^3 P (1 - e^2)^{3/2}$$

$$= 1.0737 \times 10^{-3} \frac{(a_1 \sin i)^3}{P^2}, \qquad (1.2)$$

where $*1$ (pronounced 'star 1') is the observed star and $*2$ the unseen component. Units are: K_1 in km/s, P in days, $a_1 \sin i$ in light-seconds and masses in Solar units. The inclination is not measurable for spectroscopic orbits because we have information on the motion in

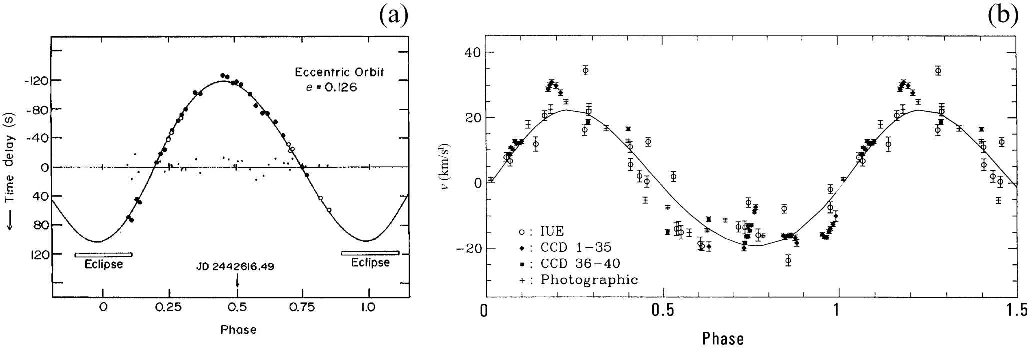

Figure 1.3 Radial velocity curves of both components of the massive X-ray binary Vela X-1 (GP Vel). (a) Doppler shift of the pulses of the X-ray pulsar: note the accurate fit to the Keplerian curve ($P = 8.964$ days, $e = 0.126$, $f_1 = 18.5\ M_\odot$). Small dots near the axis are the residuals multiplied by 2. (b) Doppler shift of absorption lines in the visible spectrum: note the larger scatter, due to irregular pulsations. From these lines $f_2 \sim 0.013$. The ratio f_2/f_1 is the cube of the mass ratio q (~ 0.09). (a) is from Rappaport *et al.* (1976), (b) from van Kerkwijk *et al.* (1995b).

only one dimension, the line of sight, whereas in visual binaries we have information in two dimensions, both perpendicular to the line of sight. In fact the red giant in ξ^1 Cet (Fig. 1.1b) does show orbital motion ($P = 1642$ days, $e = 0$, $f = 0.035\ M_\odot$, Griffin and Herbig 1981) in addition to being a composite-spectrum binary.

The mass function represents the minimum possible mass for the unseen star, which would be achieved in the somewhat improbable case $M_1 = 0$, $i = 90°$. Slightly more realistically, we might replace $\sin^3 i$ by its average value $3\pi/16 \sim 0.59$ if i is distributed uniformly over solid angle. However the value 0.59 is likely to be an underestimate, because the mere fact that a variation in radial velocity is seen implies that the lowest inclinations can be rejected. For a large ensemble of binaries we might make statistical estimates using a maximum-likelihood procedure. However, for an isolated system, with little else to guide us, we will commonly assume that a reasonable estimate of the reciprocal of $\sin^3 i$ is 1.25. We then take

$$M_1 \sim 1.25 q (1+q)^2 f_1, \tag{1.3a}$$

$$M_2 \sim 1.25 (1+q)^2 f_1, \tag{1.3b}$$

where $q \equiv M_1/M_2$ is the mass ratio. Sometimes we can estimate M_1 directly from the spectrum of the star, which may be similar to stars whose masses are already known from more favourable binaries (see below); then from Eq. (1.3a) q can be estimated and hence M_2. Alternatively one can often infer that $q > 1$ simply from the probability that the unseen star is less massive than the visible one. In either case both masses could be *considerably* greater than the mass function.

If the system is 'double lined' ('SB2'), and both components have measurable radial velocity variations (Fig. 1.3), we can further obtain the mass ratio, and hence the two quantities $M_1 \sin^3 i$ and $M_2 \sin^3 i$; but we still have no information on i. However, some SBs with $P \gtrsim 1$ year are also VBs, and in favourable cases all four of M_1, M_2, i and D can be separately measured, D in such cases being *independent* of parallax (which may be too small to be measurable).

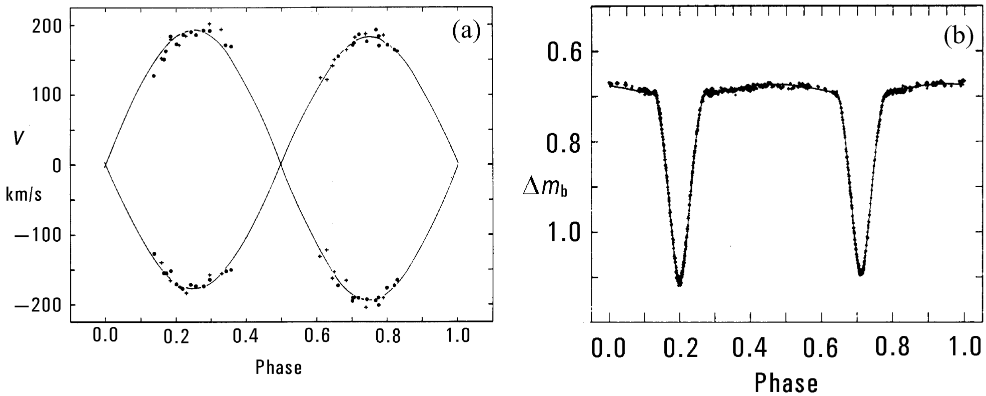

Figure 1.4 (a) The radial velocity curves and (b) the light curve of the eclipsing SB2 system V760 Sco ($P = 1.73$ days). The two components are nearly but not quite identical: in (a), $*2$ has a slightly greater velocity amplitude, and in (b) the second eclipse is slightly shallower than the first. An 'ellipsoidal variation' is seen in the nearly flat portions between eclipses. From Andersen *et al.* (1985).

Among SBs we can include both radio and X-ray pulsars, because the rapid pulsations of these objects, due to rapid rotation of an obliquely-magnetised neutron star, are often very stable and so can reveal a variable Doppler shift due to Keplerian orbital motion. Commonly, pulsar orbits are much more accurate than SB orbits based on spectral lines, so that even companions of terrestrial planetary mass can be detected (Wolszczan and Frail 1992). The much greater accuracy of radio pulsar orbits means that a number of relativistic corrections to Keplerian orbits can be measured (Taylor and Weisberg 1989, Backer and Hellings 1986). Two of these are (a) the rate $Z_{\rm GR}$ of advance of periastron in an eccentric orbit due to general relativity – Appendix C(a):

$$Z_{\rm GR} = \frac{3G(M_1 + M_2)}{c^2 a(1 - e^2)} \frac{2\pi}{P}, \tag{1.4}$$

and (b) a combination γ of gravitational redshift and transverse Doppler shift:

$$\gamma = \frac{G(M_1 + 2M_2)e}{c^2(M_1 + M_2)} \frac{P}{2\pi}. \tag{1.5}$$

Along with the mass function Eq. (1.2), these two quantities allow one to determine all three of M_1, M_2 and i, even although the orbit is 'single lined'.

X-ray pulsar orbits, though commonly more accurate than radial-velocity orbits from spectral lines (Fig. 1.3), are also commonly less accurate than radio pulsar orbits, because the X-rays come from accretion of gas lost by the companion. The gas flow is normally not steady, and so the neutron star's spin rate is erratically variable by a small amount.

Photometric binaries are stars whose light output varies periodically, and in a manner consistent with orbital motion. Usually they show eclipses, but in some cases where the inclination does not permit an eclipse one may nevertheless recognise 'ellipsoidal variation' or the 'reflection effect' (see below). A light curve (Figs. 1.2b, 1.4b) can yield, in favourable circumstances, P, e and i, the ratios R_1/a, R_2/a of stellar radii to orbital semimajor axis, and the temperature T_2 provided that T_1 is known already, from a spectroscopic analysis of the brighter component. The radius ratios and i come primarily from the duration and shape of the total and partial segments of the eclipse, and the temperature from the relative depths of the deeper and shallower eclipse in each cycle. Although some light curves can be analysed crudely by assuming that both stars are spheres, the majority of eclipsers need more sophisticated modelling, usually assuming that both components fill equipotential surfaces of the combined gravitational and centrifugal field of two orbiting point masses (the Roche potential, Chapter 3). Such light curve analysis was pioneered by Lucy (1968), Rucinski (1969, 1973), and Wilson and Devinney (1971). Information on 3546 eclipsing binary stars is given in the catalogue of Wood *et al.* (1980). A catalogue by Budding (1984) gives light curve solutions for 414 eclipsers.

An eclipsing binary is also usually a spectroscopic binary, but not conversely. This is because eclipses are only probable in systems where one star's radius is $\gtrsim$10% of the separation, whereas there is no such limit on radial-velocity variations. In the best cases, where the system has eclipses and is also double lined ('ESB2', as in Fig. 1.4), we can hope to obtain all of the following fundamental data: P, e, i, a, M_1, M_2, R_1, R_2, T_1, T_2 and D (independent of parallax). The last three of these quantities depend not only on good orbital data but also on reliable modelling of stellar atmospheres, so that the effective temperature of at least one component (presumably the brighter) can be determined directly from its spectrum. This is probably reasonable for the majority of stars, but for extremes of effective temperature and luminosity (O and M stars; supergiants and subdwarfs), spectra may be affected by such difficulties as mass loss, instability, convection and metallicity, all of which are not yet well understood. A comprehensive review of data for ESB2 binary stars in the main-sequence band has been given by Andersen (1991); an earlier review by Popper (1980) also gave data for some post-main-sequence binaries. Accuracies of $\lesssim$2% for all quantities are achievable in favourable cases.

Binaries involving evolved stars (giants, supergiants, hot subdwarfs, white dwarfs, etc.) are relatively rare, especially ESB2 systems. Although the photometric and spectroscopic data may be of the same quality, or even better, it is difficult to achieve the same accuracy in the estimation of radii. This is because the two radii are of course very different in giant/dwarf binaries. The information on relative radii, as well as on inclination, is contained in the shape of the ingress/egress portions of eclipses. If one star is so much larger than the other that its occulting edge is virtually a straight line, then the inclination and hence also the ratio of radii are indeterminate. Nevertheless supplementary information from model atmospheres, and from spectrophotometry, the measurement of intensity in several wavebands that may extend from UV to IR, can reduce the indeterminacy. Recent work on such 'ζ Aur' systems (Schröder *et al.* 1997) gives parameters with sufficient accuracy that theoretical models of stellar evolution are seriously tested.

The fact that ESB2 binaries can in principle give a distance measurement that is independent of parallax implies that they could be good yardsticks for measuring distances to external galaxies. Current and developing technology means that at least OB-type binaries may be accessible in fairly nearby galaxies. Of course one does need an estimate of the metallicity in order to relate measured colours to the effective temperature of at least the hotter component.

Because stars in close binaries can be distorted from a spherical shape by the combined gravitational and centrifugal effect of an orbiting close companion, they may show a measurable light variation even when they do not eclipse. This is called 'ellipsoidal variation' – although the stars are only approximately ellipsoidal. Figure 1.4b shows this variation. The system illustrated is in fact at an inclination which also allows eclipses: the ellipsoidal

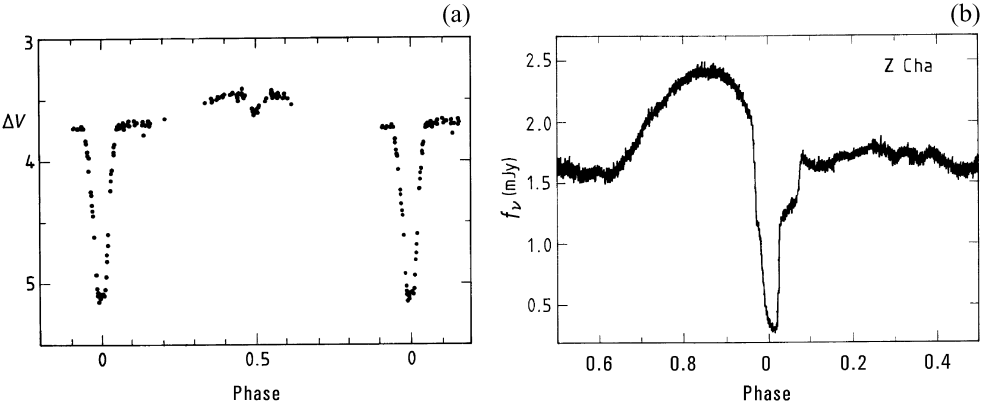

Figure 1.5 (a) The light curve of UU Sge ($P = 0.465$ days), the central star of the planetary nebula Abell 63. The hump centred on the secondary eclipse is due to a 'reflection effect'. The fainter, cooler companion shines partly by reprocessed UV light from the very hot companion; thus it is brightest just before and after it is eclipsed, and is rather faint for half the orbit. From Bond *et al.* (1978). (b) The light curve of Z Cha, an ultra-short-period binary containing a white dwarf and a red dwarf ($P = 0.0745$ days). The hump before the eclipse, the double-stepped nature of the eclipse, and the erratic variation are all due to streams of gas flowing from the red dwarf towards, and round, the white dwarf. From Wood *et al.* (1986).

variation is the slight curvature visible between the eclipses. Such variation even in the absence of eclipses may allow at least P to be determined. Further, if $*1$ (say) is much hotter than $*2$, the hemisphere of $*2$ facing $*1$ may be substantially brighter than the other hemisphere, leading to an orbital variation (Fig. 1.5a) that also does not necessarily involve an eclipse. This is called the 'reflection effect' – although the light (or X-radiation, in some cases) is absorbed, thermalised and reemitted, rather than reflected.

However, not all eclipse light curves, even with high signal-to-noise ratios and with modern light-curve synthesis techniques, lend themselves to accurate measurement of fundamental data (masses, radii, etc.). Neither do all radial velocity curves, even when a non-uniform temperature distribution over the stellar surfaces due for example to the reflection effect is allowed for. This is because stars which are close enough together to have a reasonable probability of eclipse (typically, $R_1 + R_2 \gtrsim 0.2a$) are also quite likely to interact hydrodynamically and hydromagnetically, introducing the complications of gas streams, and of starspots, which are hard to model in any but an ad hoc manner. Figure 1.2b shows a light curve of a contact binary that changed appreciably over time. The changes, and slight asymmetry, can be attributed to transient starspots. Figure 1.5b shows the light curve of a dwarf nova: an eclipse of sorts is clearly recognisable, but the light variation outside the eclipse is due to gas which streams from one component into a ring or disc about the other. Modern methods of analysis such as eclipse mapping (Horne 1985, Wood *et al.* 1986) and Doppler tomography (Marsh and Horne 1988, Richards *et al.* 1995) use image-processing techniques based on maximum-entropy algorithms (Skilling and Bryan 1984). The object of eclipse mapping is to reconstruct the distribution of light intensity over (in the case of Z Cha, Fig. 1.5b) a hypothesised flat, rotating disc of gas around one star that is fed by a stream that comes from the other star. The eclipsing edge of one star as it moves across the disc and stream helps to locate the hotter and

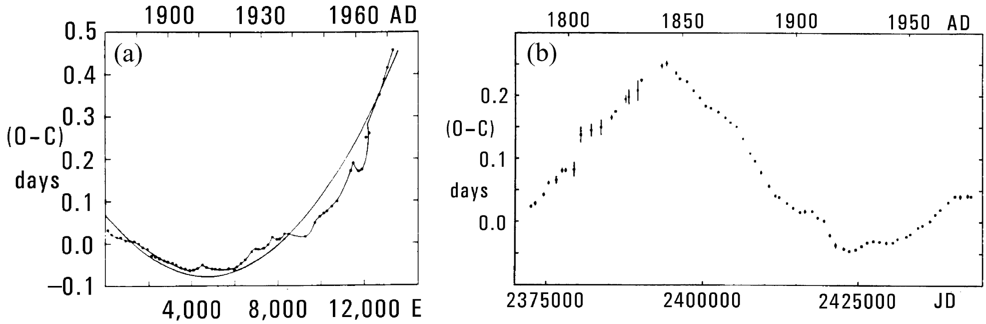

Figure 1.6 Observed times of eclipse, minus computed times obtained by assuming a constant period, plotted against cycle number (epoch) along the bottom and date along the top. (a) U Cep (G8III + B7V; 2.5 days), from Batten (1976). (b) β Per (G8III + B8V; 2.9 days), from Söderhjelm (1980). U Cep shows small erratic variations superimposed on a long term trend of increasing period; β Per also shows erratic fluctuations, but with no clear long-term trend.

cooler parts of the flow. In Doppler tomography, high wavelength resolution across a spectral line, combined with high time resolution, gives a map of intensity on a two-dimensional space of wavelength and orbital phase. This can in principle be Fourier-inverted to map intensity onto a two-dimensional velocity space, and from there one can go via some hypothesised model to a distribution in two-dimensional coordinate space. This might be either a disc-like structure, as in Z Cha, or a distribution of spots over a spherical surface, or even of spots over the joint surface of two stars that are so close as to be in contact (Bradstreet 1985). In this way one can hope to remove the distorting effect of spots and streams from the observational data, and thus be left with accurate fundamental data. But the hypothetical models of spots and streams are not in practice very strongly constrained – for example some systems may contain hot spots as well as cool spots – and so there remains considerable uncertainty in the fundamental data for many, indeed most, interacting systems.

Much information on the statistics of eclipsing binaries (and other types of variable star) comes, as a by-product, from gravitational microlensing experiments (Paczyński 1986). If a relatively nearby star happens to pass very close to the line of sight of a distant star, the apparent brightness of the distant star is temporarily increased by gravitational focusing in the field of the nearby lensing star. Such events are rare, but have been detected by several astronomical groups who monitor photometrically a large number of stars ($\sim 10^6$) in a small area of sky at frequent intervals (e.g. nightly) over several years. The light curve of a lensing event is recognisably different from the light curves of pulsators, eclipsers, novae etc.; but a large number of normal eclipsers shows up as well, and this gives a valuable database from which the statistics of orbital periods can be improved (Udalski *et al.* 1995, Alcock *et al.* 1997, Rucinski 1998). A very few lensing events also exhibit binarity directly: if the lensing object is binary it can produce a marked characteristic distortion on the light curve of a lensing event (Rhie *et al.* 1999).

Some binaries, particularly eclipsing binaries, show a measurable change of period over substantial intervals of time. Period changes are usually demonstrated by 'O − C diagrams' (Fig. 1.6). The difference between the observed time of eclipse, and the computed time based on the assumption of constant period, is plotted as a function of time (or of epoch, i.e. cycle number). One can hope by this method to determine the rates of evolution due to mass transfer or angular momentum loss.

Sometimes the change is periodic. Two possible causes of periodicity (apsidal motion, and a third body) are discussed briefly below. After subtracting such periodic motion if necessary, remaining changes might be an important indication of long-term evolution in the system. But often the long-term behaviour is contaminated by, or even completely obscured by, short-term irregular changes. Figure 1.6a shows the O − C curve for U Cep over the period 1880–1972. If the period were constant we would expect a straight line, and if the period were changing at a constant rate we would expect a parabola as shown. It can be seen that the overall behaviour is roughly parabolic, but with fluctuations of $\sim$1–2% of the period ($\sim$0.05 days) that are not attributable solely to measuring uncertainty. From the parabolic trend we infer $t_{\rm P} \equiv P/\dot{P} \sim 1.3$ megayears. The fluctuations are probably due to changes in the distribution of hot luminous gas in this unusually active Algol-like system (Olson 1985). Figure 1.6b shows the same diagram for Algol (β Per) itself over the last 200 years. Unlike U Cep, there is no clear underlying trend: only fluctuations, with possibly the same origin as for U Cep, superimposed on what appears to be a rather sudden period decrease ($\Delta P/P \sim -2 \times 10^{-5}$) around 1845, and a subsequent rather smaller and less sudden period increase around 1920.

O − C curves *ought* to be an important tool for the investigation of the slow changes expected as a result of evolution. One does not have to wait a million years in order to measure a $t_{\rm P}$ of say 10^8 years quite accurately. If the trend is clearly parabolic, and if individual eclipse timings are accurate to $\pm\delta t$, then we only need observations over a time interval Δt where

$$\Delta t \sim 10\sqrt{\frac{|t_{\rm P}|\delta t}{X}}, \tag{1.6}$$

to determine $t_{\rm P}$ to $\sim X\%$. If the eclipses can be timed to one-minute accuracy, then in a century we can hope to determine an evolutionary timescale of $\sim 10^8$ years reasonably accurately. Unfortunately, rather few binaries show anything like a consistent parabolic trend; we are not helped by the fact that a portion of a parabola can also look like a portion of a periodic third-body effect. If we had observed it only over the last century, β Per might have seemed to show a reasonable parabolic trend. However, the previous century showed quite different behaviour.

Some O − C curves show a clear periodic behaviour that can be attributed to the presence of a distant third body. AS Cam, Fig. 1.7a, is an example, although in this case somewhat marginal. The variable light-travel time due to motion round the third body causes a periodic advance/delay in the eclipse, much as the pulsar orbit in GP Vel (Fig. 1.3a) causes a periodic advance/delay in the arrival time of X-ray pulses. However, orbits of third bodies found by O − C curves are usually very long: an amplitude of 0.1 days translates into an orbital size of about 0.1 light-days or 20 AU, and so a period of $\sim$100 years. Such orbits should not be considered reliable unless at least two full orbits have been followed; of course the same qualification applies to any radial-velocity orbit, except for some radio-pulsar orbits where timing can be extraordinarily accurate. In fact Algol itself has a third body in a 1.86 year orbit, but this would not show up in the noise of Fig. 1.6b, even if plotted on a finer scale.

The timing of eclipses is also affected by ‘apsidal motion’. The gravitational force between the stars may not be a pure inverse-square law, because (a) general relativity gives a slightly different force and (b) stars can be distorted from the spherical, partly through rotation and partly through the gravitational field of the companion. The line of apses (i.e. major axis)

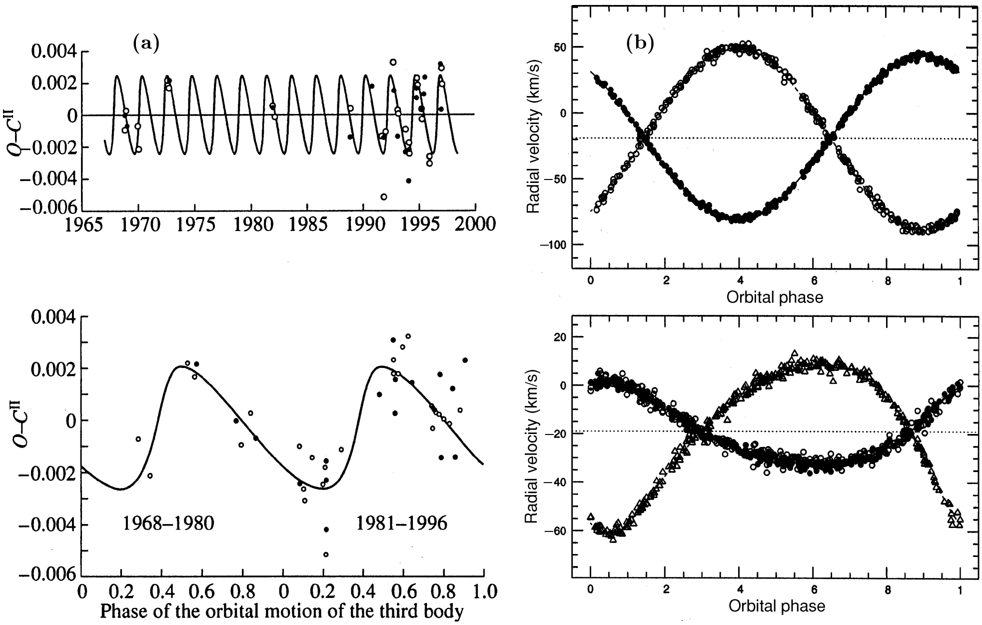

Figure 1.7 (a) O − C curves for the eclipsing binary AS Cam, as a function of date (upper panel) and of phase (lower panel). It shows a roughly periodic variation which may be due to the presence of a third body, in an orbit with $P = 805$ days, $e \sim 0.5$, $f \sim 0.03\,M_\odot$. The inner orbit has $P_\mathrm{I} = 3.43$ days, $e_\mathrm{I} = 0.17$, $(M_{11}, M_{12}) = (3.3, 2.5)\,M_\odot$. After Kozyreva and Khalliulin (1999). (b) The radial velocity curves for the inner and outer orbits of HD109648. Parameters are $P_\mathrm{I} = 5.48$ days, $e_\mathrm{I} = 0.01$, $(M_{11}, M_{12}) \sin^3 i_\mathrm{I} = (0.67, 0.60)\,M_\odot$ for the inner orbit (upper panel) and $P = 120.5$ days, $e = 0.24$, $(M_1 \equiv M_{11} + M_{12}, M_2) \sin^3 i = (1.09, 0.54)\,M_\odot$ for the outer (lower panel). From Jha *et al.* (2000).

of a Keplerian orbit is only fixed in space if the force is *exactly* inverse square. Departures make it rotate, and if the orbit is eccentric this means that the eclipses will vary periodically, particularly in the orbital phase of one eclipse relative to the other. The rate of rotation of the line of apses can be measured, and used to check models of internal structure. The rate has also been perceived as a test of GR, but since GR has been verified (see below) to very great accuracy any explanation of discrepancies has to be sought elsewhere.

For example, AS Cam (Fig. 1.7a) shows apsidal motion at a rate inconsistent with GR. Probably this is due to the third body, which affects the apsidal motion as well as introducing a periodic delay (Kozyreva and Khalliulin 1999). Apsidal motion shows up as a slight difference in the period, depending on whether one follows the primary (deeper) eclipse or the secondary. This is because as the major axis rotates slowly the interval between the primary and secondary eclipse changes. Ultimately, the behaviour should be cyclic, with an estimated period (for AS Cam) of ~2400 years. The difference in period has however already been allowed for in Fig. 1.7a, where primary eclipses are denoted by heavy dots and secondary eclipses by circles. What remains is not quite constant, but shows (marginally) a periodic fluctuation with an amplitude of ~0.002 days and a period of ~2 years. This is arguably the 'light-time effect' of a third body, which like a radial-velocity curve (also a Doppler effect) gives a mass function as well as period and eccentricity as listed in the figure caption.

The inconsistency noted above between the measured and theoretically estimated apsidal motion may be due to this third body. Such a body can inject additional apsidal motion (of either sign) into the system, which – somewhat coincidentally – could be of the same order of magnitude (Appendix C).

Figure 1.7b illustrates the radial velocity curves that can be obtained in favourable circumstances from a triple system, HD109648. The spectrum is composed of three separate F stars, two of which show rapid cyclic variations and the third a slower cyclic variation. Not just three but four radial velocity curves can be determined: one is the motion of the centre of gravity of the short-period pair, and mirrors the motion of the third, slowly-moving, spectrum. This gives four mass functions, but unfortunately there are five unknowns: three masses and two inclinations.

Radio pulsars allow enormously greater accuracy to be achieved (Taylor and Weisberg 1989). Some with $P \lesssim 0.4$ days demonstrate the very slow period decrease expected from GR, on a timescale of $\gtrsim 10^8$ years (Section 4.1). For PSR 1913 + 16, the theoretical rate agrees with the observed rate to within one per cent, which is the observational uncertainty. Pulsars near the centre of a globular cluster even show acceleration due to the cluster's gravitational field, and not just a binary companion. What a pity that most stars do not have a pulsar companion!

1.3 Stellar multiplicity

Although only a few thousand stars are well established as binary, with known orbital periods, the incidence of binarity among the most thoroughly observed stars (generally the brightest, but also the nearest) is very high. Conceivably all stars are binary, or of even higher multiplicity. We normally think of the Sun at least as being single, but if there is a continuum of objects from small planets like the Earth ($\sim 3 \times 10^{-6}\, M_\odot$), through massive planets like Jupiter ($0.001\, M_\odot$), to small stars, then perhaps the distinction between single and binary is artificial. Recently detection sensitivity and strategy have improved to the point that three Earth-mass companions to a pulsar (PSR 1257 + 12; Wolszczan and Frail 1992) have been found, and Jupiter-mass companions to about 100 nearby stars, mostly of Solar spectral type (Mayor and Queloz 1995, Marcy and Butler 1998).

A common definition of the term 'star' is that it is an object with mass greater than $\sim 0.08\, M_\odot$, because this is the minimum mass for a self-gravitating hydrostatic spherical gaseous body that can support its radiant energy loss by hydrogen fusion. However, this is a somewhat artificial boundary, because stars in the process of forming will not 'know' that they may come up against this distinction. Low-mass dwarfs are known whose masses are only just above the limit, for example UV Cet (Gl 65AB), a VB where both components are late M dwarfs of $\sim 0.11\, M_\odot$ (Popper 1980). Objects below the critical mass but well above Jupiter's mass are referred to as 'brown dwarfs'. Some are known to exist, but they are hard to detect. An example of a binary containing a star so cool and faint that it is almost certainly below the critical mass is Gl 229AB (Nakajima *et al.* 1995). Some recent low-amplitude orbits of Solar type stars (e.g. HD140913, Mazeh *et al.* 1996) point to companions of $\lesssim 0.05\, M_\odot$, though of course with the ambiguity that the inclination can only be guessed, i.e. assumed not to be improbably small. Observations in the IR (Rebolo *et al.* 1995) have recently been turning up a wealth of probable brown dwarfs in, for instance, the Pleiades cluster.

Recent SB1 detections of companions down to about a Jupiter mass suggest a bimodal distribution, with a fairly rapid drop in numbers to lower mass in the range 0.3–0.07 $M_\odot$, a low plateau in the brown-dwarf region 0.07–0.01 $M_\odot$, and then a peak for major planetary masses below $\sim$0.01 $M_\odot$ (Marcy and Butler 1998). This is consistent with the likely hypothesis that the formation mechanism of binary stars is very different from that of planetary systems. The two processes are not exclusive, however. Some systems are known to have both a planetary companion *and* a stellar companion: τ Boo (Butler *et al.* 1997, Hale 1994), 16 Cyg (Cochran *et al.* 1997) and υ And (Lowrance *et al.* 2002). The last has three massive planets and a distant M-dwarf companion.

Most stars are members of binaries. Petrie (1960) showed that 52% of a sample of 1752 stars, independent of spectral type, have variable radial velocities. Since not all short-period binaries can be detected due to finite measuring accuracy, it follows that substantially more than 50% of stars are in relatively short-period binaries. After considering unseen companions, Poveda *et al.* (1982) concluded that nearly 100% of stars are in binaries, including long as well as short periods.

For the sake of terminology, we assume here that there *are* such things as single stars, distinct from binary stars. In other words, we accept the presently-known multiplicity of a particular system, not withstanding the possibility, even probability, that more detailed measurement will mean that small or distant companions will be detected. Thus if a star is not presently known to have a companion, we will speak of it as single. Furthermore, if there is a binary companion but it is too far away ever to have an effect on the evolution of the target star, we shall often use the term 'effectively single'.

Many systems once thought to be binary turn out to contain three or more stars. According to Batten (1973), double-star systems are roughly twice as common as single-star systems, but for $2 < n \leq 6$ the number of systems containing n stars falls off very roughly as 4^{-n}. This means that $\sim$25% of all systems, and $\sim$15% of all stars, are single, while $\sim$20% of all systems ($\sim$30% of stars) are in triples or higher multiples; the average system contains about two stars. Duquennoy and Mayor (1991) found a slightly lower incidence of multiplicity in a sample of 161 F/G-type systems: 92 single, 61 binary, 6 triple and 2 quadruple, but with a proviso that 18 components in this sample showed significant radial velocity variations that might indicate further multiplicity. Tokovinin (1997) has catalogued 612 triple and higher-multiple systems. In this book we will make the assumption, when illustrative numbers are necessary, that $\sim$30% of systems are single to present levels of accuracy, $\sim$60% are binary and $\sim$10% are at least triple.

The incidence of multiplicity is probably not independent of the kind of star being sampled. The 42 nearest stellar systems (within $\sim$5 pc; excluding the Sun itself) are mostly M dwarfs, with less than half the mass of the Sun. They contain at least 14 multiples – 10 binary and 4 triple. They also contain at least one massive planet, around an M dwarf star. On the other hand, the 48 brightest systems ($V \leq 2.0$; from Hoffleit and Jaschek 1983 and Batten *et al.* 1989) are mostly B and A stars, typically more than twice the mass of the Sun. They contain at least 22 multiples – 14 binary, 3 triple, 4 quadruple and 1 sextuple. The statistics are not compelling, of course, but seem to imply that more massive systems are more highly multiple. For both these samples, small as they are, the data are far from complete, and the actual multiplicity could well be higher.

It is always difficult to compare distance-limited samples of stars with magnitude-limited samples, because binaries are inherently brighter than single stars, although not by much unless the masses are fairly closely equal. Obviously the first kind of sample is to be preferred where possible, but distances are much harder to measure than magnitudes. In the above two samples the effect is probably quite small.

Multiple ($n > 2$) systems tend to be 'hierarchical' (Evans 1968, 1977), i.e. they consist for example of two close 'binaries' whose centres of gravity rotate around each other in a wide 'binary'. Such a configuration is expected to be stable on a long timescale, provided that the period of the wide 'binary' is several times greater than the period of either close 'binary'. Just how much greater the longer period must be for stability depends strongly on the eccentricities, and also the inclination of the outer orbit to the inner orbit, but for orbits which are nearly circular and coplanar it is typically in the range 3–6, assuming that all the masses are comparable. Observed systems usually have a much greater period ratio than this (10^2–10^4, and even more), and are therefore likely to be extremely stable even allowing for orbital eccentricity and non-coplanarity. Figure 1.7b shows the inner and outer orbits of the triple system HD 109648 (Jha *et al.* 2000). This system of three rather similar F dwarfs has an unusually small period ratio of about 22. One of the two velocity curves in the lower panel of Fig. 1.7b is the velocity of the centre of gravity (CG) of the inner pair.

The well-known sextuple system α Gem is a microcosm which contains within itself two VBs, two SB1s and an ESB2. Its components are organised as follows, using a notation of nested parentheses to emphasise the hierarchical nature:

$$(((\mathrm{A1V} + ?; 9.2\,\mathrm{d}, e = 0.5) + (\mathrm{A2:m} + ?; 2.9\,\mathrm{d}); 500:\mathrm{yr}, e = 0.36; 7'')$$
$$+ (\mathrm{M1Ve} + \mathrm{M1Ve}; 0.8\,\mathrm{d}); 70'').$$

The outermost orbit of the α Gem system is too slow to be measurable, but there is no doubt that the M dwarf pair is related to the other four components, by virtue of the fact that they have a common proper motion – more precisely, the M dwarf pair has almost the same proper motion as the centre of gravity of the pair of A stars. The outermost orbital period can be expected to be of the order of 10^4 years. Each A star is an SB1, one of which has a fairly eccentric orbit. The unseen companion in each SB1 is likely to be a red dwarf, although in principle it could be some other faint object such as a white dwarf. The mass functions are known (0.0013 and 0.01 $M_\odot$ respectively), but do not rule out a substantial range of masses and inclinations. The M dwarf pair is an ESB2 – with a separate variable-star name, YY Gem – and is one of the very few systems from which M dwarf masses and radii can be determined directly.

A few multiple systems are 'non-hierarchical': three or more stars are seen which are all at comparable distances from each other. This *could* be simply a projection effect, but the probability is not large. If it is not due to projection, then such systems cannot be stable in the long run, and indeed the few that are known are groups of young stars, such as the Trapezium cluster in Orion, that have simply not yet had time to break up. If N stars of total mass M are fairly uniformly distributed in a volume of radius R, we expect the system to break up in a time comparable to the 'crossing time' $\sqrt{R^3/GM}$. The final product will typically be a series of ejected single stars, and a remaining close binary; but we might have one or more binaries ejected or a hierarchical triple left over. Usually the stars ejected will be the less massive members, and the remaining binary is likely to contain the two most massive stars.

Although most of this book is concerned with systems of only two components, there are many triple systems and a few quadruple systems known where *all* the components are close enough to interact at some stage. For the most part, when we mention a binary we are thinking of only those binaries at the bottom of the hierarchical pyramid: three binaries in the case of α Gem.

1.4 Nomenclature

In discussions of binary and (necessarily) more highly multiple stars, we should probably be careful to use the word 'system' rather than 'star', since the latter term is often ambiguous – commonly used to mean either the individual components, or alternatively the whole ensemble of components. We should also be cautious about using the words 'primary' and 'secondary': some authors use 'primary' to mean the more luminous star (at least in a particular wavelength range), others to mean the hotter star, and others still to mean the component that has the lowest right ascension.

When discussing the behaviour of a binary, it will often be convenient to refer to the components as $*1$ and $*2$ – to be pronounced 'star 1' and 'star 2'. I will use two somewhat contradictory conventions throughout this book, depending on context. Sometimes, for example when discussing an SB1 binary, $*1$ will mean the star we see and $*2$ the star we do not see, as in Eq. (1.2). At other times, for instance when discussing an SB2 binary, I will take $*1$ to be the component which we infer to have been *initially* the more massive, and $*2$ to have been *initially* the less massive. For example, in Fig. 1.1a $*1$ would be the F5V star and $*2$ would be the G5V star; in Fig. 1.1b $*1$ would be the white dwarf and $*2$ the G8III star. When I am discussing the theoretical behaviour of one component of a binary, in Chapter 3 and later, I will usually call that component $*1$, often considering, for illustrative purposes, that $*2$ is just a point mass with no structure. But when I am discussing the long-term evolution of both components of a binary, I will adhere again to the principle that $*1$ is the initially more massive component. I do this because any discussion of the evolution demands that we identify the same component as $*1$ throughout several substantial changes in mass ratio, luminosity ratio, etc. I do not apologise for possible confusion because I consider that there is no convention which (a) I could adhere to rigorously and (b) would not cause confusion at some point.

It may appear that we are bound to have even less information on *initial* masses than on *current* (measurable) masses. However, this is really not so, given the rather clear theoretical understanding (Chapter 2) that the rate of evolution of a star is principally determined by its initial mass, rather than its current mass, provided only that mass loss or gain did not begin at a *very* early stage in the life of the star. If we see a combination of white dwarf and red giant we can be reasonably sure that the white dwarf is $*1$ in the second sense defined above, even though we may have little or no information on the *current* masses. Only a modest fraction of binaries poses any real challenge to this assumption.

A further convention that I will impose throughout most of this book is that the mass ratio q is M_1/M_2, rather than its reciprocal. Thus, in the *second* convention above, $q \geq 1$ at zero age – but at a later stage of evolution q may drop below unity because of evolutionary changes of mass in one or other (or both) components. However, in this chapter alone, I will use Q defined as $1/q$. In this preliminary discussion I concentrate mainly on unevolved binaries, where the brighter and hotter component can be reasonably assumed to be $*1$. In such systems the mass ratio M_2/M_1 is usually what is discussed in the literature, and this is Q, not q. In Chapter 3 and later, however, Q is a dimensionless quadrupole moment of a distorted star.

In a hierarchical triple system logic demands that I refer to the outermost pair as $*1$, $*2$, and the inner pair (if it is $*1$ that is binary) as $*11$, $*12$. The periods would be P for the outer pair and P_1 for the inner pair. By extension, in α Gem above the unseen close companion to the A2:m star is $*122$, for example, and $P_{12} = 2.9\,\mathrm{d}$. However, logic does not quite dictate which of the two A-type SB1s is $*11$ and which is $*12$:

(a) the more massive *pair*?
(b) the pair that has the most massive component?
(c) the more highly multiple (supposing that the multiplicities were different)?

I shall duck this issue in its fullest generality. For triples, I shall

(a) attempt to identify the *originally* most massive of the three components, as argued above for binaries, and
(b) name the subsystem that contains it as $*1$, which might be either single or binary.

I have used this convention in the caption of Fig. 1.7, regarding the two triple systems AS Cam and HD109648. The data given there for the second system show that the two orbits within it are not parallel to each other: $\sin i_1 = 1.06 \sin i$. Unfortunately even if $\sin i_1 \sim \sin i$ we cannot assume that the orbits are parallel. In the system β Per referred to previously both the inner (eclipsing) orbit and the outer (visual) orbit are inclined at nearly 90° to the line of sight. But radio interferometry with a very long baseline (VLBI) shows that the orbits are actually inclined to each other at ∼100° (Lestrade *et al.* 1993). Nature appears to be playing a rather cruel joke, since the probability that the two orbital axes and the line-of-sight axis are all (nearly) mutually orthogonal must be rather small.

Although Nature is probably logical most of the time, the human perception of it is often influenced by customs that are historical and cultural rather than logical. Consequently, we shall in practice refer quite often to '$*3$' and even '$*4$', as distant companions to some binary of interest, or as recently-discovered close companions to components of a wide binary. It is unfortunate but unavoidable that whenever a new component is discovered the names of some at least of the previous components will have to change.

1.5 Statistics of binary parameters

The statistical distributions of masses, orbital periods, mass ratios and eccentricities are not well known: see for example discussions by Heintz (1969), Griffin (1983, 1985), Zinnecker (1984), Trimble (1987), Halbwachs (1983, 1986), Hogeveen (1990), Duquennoy and Mayor (1991) and Halbwachs *et al.* (2003). That many systems (perhaps $\gtrsim$10%) are at least triple makes it even harder to arrive at a firmly-based distribution of these parameters. I concentrate in this book primarily on systems whose periods are short enough to allow for some kind of binary interaction.

1.5.1 Binary interaction

Although most stars are as small as the Sun, or smaller, they are capable of growing in radius by a factor of ∼1000 during their evolution (Chapter 2; Table 3.2). The Sun may well fill the orbit of Mars or even Jupiter before collapsing to a white dwarf. A substantially more massive star than the Sun could grow to an even larger radius, before exploding as a supernova. Only if the period is longer than $\sim 10^4$ days (∼30 year) is there a reasonable probability that the two components go through their entire evolution almost independently (Plavec 1968, Paczyński 1971). However, some interaction (in addition, of course, to the basic gravitational one) can take place in even wider systems. The prototype Mira variable *o* Cet has a white-dwarf component (VZ Cet, $*1$) in a roughly 400 year orbit about the M supergiant pulsating variable ($*2$). The white dwarf flickers rapidly, unlike normal single white dwarfs, and this is probably because it is interacting with the copious wind that is being ejected by the Mira. It may be accreting only a small fraction of this wind, but that could be enough to affect the white dwarf's future evolution.

Although stars in systems with orbital periods substantially in excess of 100 years are not likely to undergo very much mutual interaction, this is not to say that their orbits remain uninterestingly constant for all time. A supernova explosion, or the ejection of large amounts of gas by a blue or red supergiant, may change or even disrupt the orbit. So also can the random perturbations imposed by interaction with nearby systems. This last effect imposes a loose upper bound on the orbital separation of individual systems. The orbital separation can hardly be larger than the mean distance between independent systems, $\sim$1 pc in the Solar neighbourhood, and this translates by Kepler's law into an upper limit on orbital period of $\sim 10^{10}$ days for a system of mass $\sim 1\ M_{\odot}$. In practice the upper limit is likely to be at least an order of magnitude less, since many near collisions of such a system with adjacent systems can be expected in the course of the Galaxy's lifetime.

Within a dense cluster of stars, such as a globular cluster, and also near the Galactic centre, it is possible for binaries of shorter period to be disrupted by near collisions. It is even possible in such an environment for binary stars to be *formed*, for example by tidal capture. This can happen if, in a close approach of two single stars, large tides are raised on at least one. Such tides can dissipate energy, and so allow the stars to move from a hyperbolic to an elliptical orbit (Fabian *et al.* 1975). Thus some interesting binaries in globular clusters need not be the products of long-term evolution of a primordial binary. In a dense stellar environment binaries can also, and in fact more easily, be modified by 'exchange' interactions, where one star in a near collision with a pre-existing binary may eject one component and replace it, perhaps in a much closer orbit. In the bulk of our Galaxy, however, such interactions are not likely because of the low stellar density, and so it is reasonable to suppose that a star which is presently binary has always been binary.

Several mechanisms are identified in Chapters 3–6 that can result in 'mergers', two components of a binary becoming merged into one. Thus the mere fact that an observed star presently appears to be single does not exclude the possibility that formerly it was binary. By extension, a system which is now a binary (but presumably a fairly wide one) may be a former triple. Mergers can be the result either of slow evolution or of some rapid dynamical event.

Returning briefly to the issue of nomenclature, it is unfortunate that the terms 'close binary' and 'wide binary' can have very different meanings depending on context. To someone, say, using speckle techniques to resolve binaries in a star-forming region at a distance of 500 pc, a binary with a separation of $0.05''$ is close, if not very close. But the linear separation is $\sim$25 AU, which for present purposes makes it a rather wide, if not very wide, binary – probably too wide to interact. In this book we will generally use 'close' to mean a period of a few days, and 'wide' to mean a few years; 'very wide' will mean too wide to interact seriously, i.e. a period in excess of $\sim$30–300 years.

Recently it has become clear that the evolution of a binary can be seriously modified by the presence of a third body, even if that body is in a *wide* orbit – perhaps 10^4 years – and even if the third body is of quite low mass so that it is hard or impossible (at present) to observe. The main requirement for important interaction ('Kozai cycles'; Kozai (1962); Section 4.8) is only that the outer orbit be substantially inclined to the inner ($\gtrsim 39^{\circ}$). In a Kozai cycle the inner orbit's eccentricity fluctuates cyclically between a small and a large value, while the period remains roughly constant. The cycle time is $\sim P_{\rm outer}^2/P_{\rm inner}$, multiplied by a factor

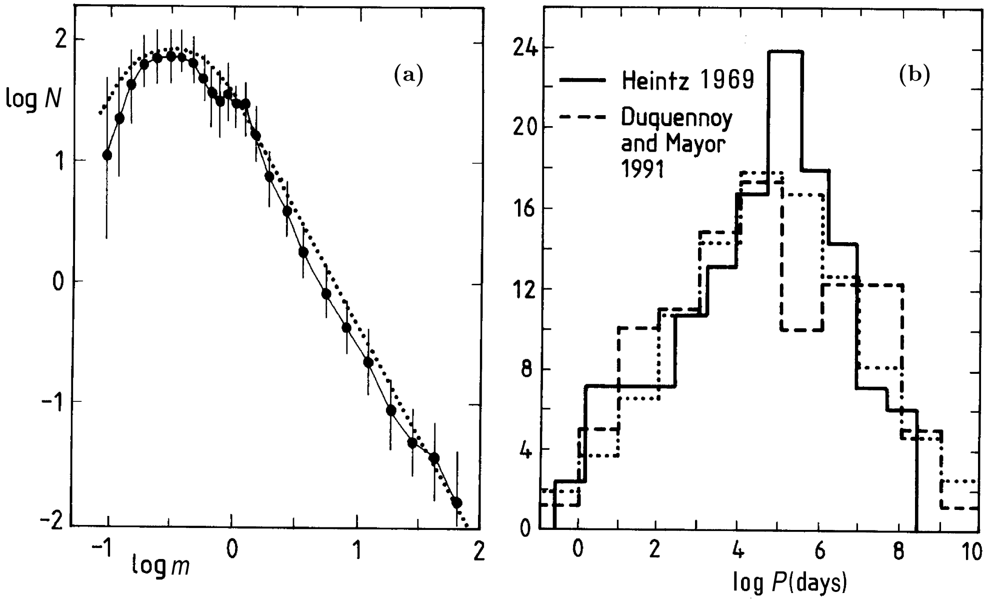

Figure 1.8 (a) The distribution of initial mass from Scalo (1986), and (dotted) the approximation of Eq. (1.10). The latter is displaced slightly upwards for clarity.
(b) Histograms of the period distribution, from Heintz (1969) (solid line), Duquennoy and Mayor (1991) (dashed line) and the approximation of Eq. (1.14) (dotted line) all normalised to the same total area.

(total mass)/(third-body mass). If the periods were 10^2 and 10^4 years, and the masses all comparable, the Kozai time would be $\sim 10^6$ years; and if the mutual inclination were 70° then the inner eccentricity would peak at 0.9, if it were zero to start with. Thus the *periastron* separation at this peak would be equivalent to a circular binary with a period of only $\sim$3 years. If the inner and outer orbits have a random inclination, a reasonable but by no means certain hypothesis for fairly wide orbits, the average inclination would be 60°, and 70° would be by no means unusual (cf. β Per, 100°, in Section 1.4). This increases substantially the scope for 'binary' interaction.

1.5.2 Masses

Stellar masses show a distribution which (per unit volume of the Galaxy) favours low masses; although, because massive stars are very much brighter than low-mass stars, the distribution down to a given *apparent* brightness favours higher masses. The Salpeter IMF (i.e. initial mass function) is the following approximation to the distribution $N(M)$ of zero-age masses as a function of mass (Salpeter 1955):

$$\begin{aligned} N\,\mathrm{d}M &\propto M^{-2.35}\,\mathrm{d}M \qquad (M \geq M_0 \sim 0.1\,M_\odot), \\ &= 0 \qquad\qquad\qquad (M < M_0). \end{aligned} \tag{1.7}$$

This distribution has to be truncated at a low mass (say $\sim$0.1 $M_\odot$), to keep the number finite. More recent IMFs (Miller and Scalo 1979, Scalo 1986, Basu and Rana 1992) show a turnover at a mass of about 0.3 $M_\odot$, as shown in Fig. 1.8a. Whether, for binary stars, a Salpeter-like

IMF is thought of as applying to *primary* mass or *total* mass is not very important, given the steepness of the IMF over most of its range. However, a careful determination of the IMF from observation ought to take into account the fact that many stars are actually at least binary (Kroupa *et al.* 1991). For the present, we suppose that so far as binaries are concerned the IMF is equivalent to the distribution of M_1, the more massive component. The main uncertainty in the IMF comes from transforming stellar apparent magnitudes and colours to absolute luminosities, and thence to masses, particularly at low mass where there is only sparse observational data from binaries on the mass–luminosity relation. But in any event an IMF contains a fair amount of theoretical input, to allow for the lifetimes of stars as a function of their masses. O stars, say 20–50 $M_\odot$, are much less abundant relative to G dwarfs ($\sim$1 $M_\odot$) than Eq. (1.7) seems to suggest, because they have lifetimes a thousand times shorter.

It is often helpful to be able to generate a distribution of some parameter by a Monte Carlo process, i.e. by use of a random number generator. Consider, for example, the Salpeter distribution of masses, Eq. (1.7). Let X be a random number chosen from a uniform distribution in the range [0, 1]. Then if we determine the mass M_1 by

$$M_1 = \frac{M_0}{(1 - X)^{0.75}}, \tag{1.8}$$

we generate the Salpeter distribution. We require $M_0 = 0.1$ if the distribution is to be truncated at 0.1 $M_\odot$, as suggested for distribution (1.7).

The physical significance of the inverse function $X(M_1)$ is that it is the fraction of all stars that have mass less than M_1, i.e. it is the *cumulative* distribution function. It may be helpful to spell out this relationship. Let us integrate and normalise the distribution (1.7):

$$X(M_1) \equiv \frac{\int_0^{M_1} N(M)\mathrm{d}M}{\int_0^{\infty} N(M)\mathrm{d}M} = 1 - \left(\frac{M_0}{M_1}\right)^{1.35}, \quad M_1 > M_0. \tag{1.9}$$

This $X(M_1)$ relation is just the inverse of the $M_1(X)$ relation (1.8), to the extent that 0.75 is approximately the reciprocal of 1.35. The mass spectrum $N(M_1)$ (normalised) is therefore just $N(M_1) = \mathrm{d}X/\mathrm{d}M_1 = 1/(\mathrm{d}M_1/\mathrm{d}X)$. A small but important point, often overlooked, is that it is better, in order to approximate an observed distribution, to start by approximating the cumulative distribution $X(M_1)$, or equivalently the inverse function $M_1(X)$, than by approximating the *differential* distribution $N(M_1)$. Coincidentally, it is also much more convenient numerically: it is usually easier to differentiate a function than to integrate it.

Believing that the Scalo (1986) distribution of Fig. 1.8a is a more accurate distribution than Salpeter's (1955), we attempt to approximate it with

$$M_1 = 0.3\left(\frac{X}{1 - X}\right)^{0.55}. \tag{1.10}$$

This mass distribution is Salpeter-like at $M_1 \gg 0.3\ M_\odot$, but with exponent 2.82 rather than 2.35. The distribution of masses generated by this formula is shown in Fig. 1.8a. It is somewhat coincidental that the slope of the mass distribution below the peak is much the same, but with opposite sign, as the slope above the peak. This allows us to use a single exponent (0.55) in the distribution (1.10). If the two slopes were markedly different one might choose

$$M_1 \propto \frac{X^\alpha}{(1-X)^\beta}, \tag{1.11}$$

but for the observational distribution illustrated this refinement seems unnecessary.

Recent work on very low-mass stars (Jameson *et al.* 2002), including spectral types L and T beyond M, suggests, although not yet with complete conviction, that the IMF continues to rise, but more slowly, below the peak of Eq. (1.10) at $M_1 = 0.3\, M_\odot$. Values of $\alpha \sim 1.5,\ \beta \sim 0.55$ in Eq. (1.12) might be somewhat better. This value of α implies that $N(M_1) \sim M_1^{-0.33}$ at low M_1, i.e. at low X.

1.5.3 Orbital periods

For spectroscopic binaries, mainly of spectral type G or K, giant or dwarf, Griffin (1985) found an increasing distribution of number N versus log P; so that, very crudely,

$$N\mathrm{d}P \propto P^{-0.7}\mathrm{d}P \propto P^{0.3}\mathrm{d}\log P, \quad (P \lesssim 30\,\text{years}) \tag{1.12}$$

over a range of periods P from days to decades, the only upper limit to period being set by the patience of spectroscopic observers (i.e. about 30 years; but one hopes this will increase). Heintz (1969) and Duquennoy and Mayor (1991) found something similar, and also found that for still longer periods, in visual rather than spectroscopic binaries, the number per decade of log P falls off again (Fig. 1.8b); the peak in the distribution occurs at roughly 200 years. For systems whose orbital periods are too long to have been measured directly, an order-of-magnitude estimate of the period can be obtained from the observed angular separation α, the distance D based on either a directly measured parallax or on spectral type and apparent magnitude (a ‘spectroscopic parallax’), and Kepler’s law, Eq. (1.1). Assuming that the sum of the masses is roughly Solar (because the observed masses range over only about two orders of magnitude while periods range over about ten), we can translate crudely but directly from separation and distance to period. The falling-off in number at longer P found by Heintz (1969) and Duquennoy and Mayor (1991) can be represented roughly by

$$N\mathrm{d}P \propto P^{-1.3}\,\mathrm{d}P \propto P^{-0.3}\,\mathrm{d}\log P. \quad (P \gtrsim 300\,\text{years}) \tag{1.13}$$

Figure 1.8b also shows the period distribution found by Duquennoy and Mayor (1991) for 79 binary periods from 161 systems that are within 22 pc of the Sun, that have an F4-G9 IV-V primary, and are north of -15°; the median period is at about 180 years.

Both the Heintz and the Duquennoy–Mayor distributions in Fig. 1.8b are fitted well, in the same spirit as Eq. (1.10), by

$$P(\text{days}) = 5.10^4 \left(\frac{X}{1-X}\right)^{3.3}, \quad (\text{F/G dwarfs}) \tag{1.14}$$

where X is a second, independent, random variable distributed uniformly over the range [0, 1]. This distribution is also shown in Fig. 1.8b. As with the mass distribution in the previous subsection, a single exponent (3.3) seems in practice to be adequate, since the slopes at short and long periods appear to be much the same but of opposite sign.

We should not assume, however, that the same distribution would be found for binaries with say OB, or M dwarf, primaries as for those with F/G dwarf primaries. In fact, the balance of short to long period systems depends markedly on primary mass. Among the 42 *nearest* systems, mostly M dwarfs, two have $P < 100$ days (excluding a massive planet in a 61 day orbit). On the other hand, among the 48 *brightest* systems (by apparent magnitude), mostly A/B dwarfs or G/K giants substantially more massive than the Sun, 10 have $P < 100$ days – although three of these are within the same sextuple system α Gem. Somewhat greater bias still towards shorter periods is shown by the 227 O-type stars with $V \le 8.0$ (Mason *et al.* 1998). These contain 52 SB orbits (23%) with $P \le 0.1$ years. This is hardly consistent with the Solar-dwarf sample of Duquennoy and Mayor (1991), where only 13 out of 161, or 8%, of systems have periods <100 days.

However, the O-star sample of Mason *et al.* (1998) shows a strongly bimodal distribution with a second, even larger, accumulation (80, or 36%) of visual binaries at estimated $P \sim 10^4$–10^6 years. This is also many more than in the G-dwarf sample. Correspondingly, there is a marked shortage of systems (37, or 16%) in the considerable intermediate range 0.1–10^4 years. It can be argued that this is the most difficult range for detection of binarity: firstly, O stars are much more distant than G stars, so they have to be further apart to be recognisable as VBs; and secondly, O stars tend to show erratically variable radial velocities at the level of ∼20 km/s, which can obscure the lower radial-velocity amplitudes in the longer-period spectroscopic orbits. Recent advances in interferometry have already increased the numbers in the 'gap', to the percentage quoted above. About 100 systems would have to be found if the gap is to be levelling off, and about 250 if it is to be turned into a modest peak as in the G-dwarf sample. These numbers are not quite as ridiculous as they sound because the high multiplicity typical of massive systems may well mean that '200% of stars are binaries'.

Nevertheless, I adopt here a relatively cautious position. Spectroscopic orbits with periods of 0.1–1 year should not be *much* harder to detect than those in the range 0.01–0.1 year. Their velocity amplitudes will be down by a factor of about 2.2, but still well above the noise level: yet only 5 are known against 33 in the shorter-period bin. The apparent shortfall might also be related to the distribution of mass *ratios*, which I discuss shortly. Perhaps low-mass companions are relatively more likely at longer than shorter periods, but a rather drastic change in the distribution of mass ratios at about 0.1 year would be required. Let us content ourselves with a distribution that peaks at about 15 days, and drops off fairly rapidly on both sides; say

$$P(\text{days}) = 15 \left(\frac{X}{1-X}\right)^{1.3} \qquad \text{(O stars).} \tag{1.15}$$

I do not suggest that this distribution can be used to include the visual binaries at very long periods, but it roughly represents the presently-known binarity among potentially interactive binaries, i.e. those with periods up to 10^2 years, giving some allowance for possible new discoveries in the period range 0.1–10^2 years. Specifically, it predicts that 50% of systems have periods over 15 days, whereas at present only 30% (of SBs) do. Our prescription also predicts that 10% of O star binaries would have periods less than 1 day, which is hardly possible given the estimated sizes of O stars; but several O stars are found with periods in the range 1.4–2.5 days. Somewhat simplistically, we will treat binaries of improbably short period from such distributions as 'merged binaries', or in other words single stars.

At the other end of the mass spectrum, for ∼200 G9–M3 dwarfs Tokovinin (1992) found an even smaller proportion (3%) of short periods than among the G/K dwarfs, let alone the O stars. It appears to be reasonable to suppose that the median period of the distribution shifts fairly continuously from short to long periods as mass decreases, and that the distribution also becomes wider and shallower. In my tentative model of Section 1.6, I suggest a distribution like Eqs. (1.14) or (1.15) but with a coefficient, and also an exponent, that is a function of mass.

The distribution of pairs of periods within triple systems is, of course, much more uncertain. I have already suggested that as a first approximation we assume that $\sim$10% of systems are at least triple. In most of these, the outer orbit will be much too wide for serious interaction as described in Section 1.5.1; but I estimate even more provisionally that $\sim$20% of triples, and thus $\sim$2% of systems, may have *both* periods shorter than $\sim$30 yr, and thus be potentially capable of two distinct interactions.

1.5.4 Mass ratios

The distribution of mass ratios is less well known than either of the distributions over period or mass. This is because substantially more orbital data are required for a mass ratio than for a mass or a period (Section 1.2). We need an SB2, rather than an SB1, for a mass ratio, and in many systems $*2$ is too faint relative to $*1$ to be measured reliably.

A common ad hoc model is made by assuming that both of the component masses are given by the same distribution, for example distribution (1.11). This is equivalent to saying that the two components have uncorrelated masses. Duquennoy and Mayor (1991) found this to be an adequate approximation for their sample of binaries whose primaries were all F/G dwarfs like the Sun. However, it cannot be an adequate approximation for massive stars, since these are intrinsically rare and yet are frequently paired with a comparably massive star. Furthermore, Lucy and Ricco (1979) found that among short-period binaries ($P \lesssim 25$ days) there is a much higher proportion of systems with nearly equal masses than can be accounted for by selection effects, strong as these are. Almost certainly the degree of correlation of the two masses is a function of both orbital period and mass, and it appears that there is more correlation at short periods or high masses.

Specifically, Lucy and Ricco (1979) found that among F2–M1 dwarfs with $P < 7.5$ days the number in the range $Q = 0.94$–1 was 50% of the number in the range 0.6–1. The Monte Carlo distribution generated by

$$Q = 1 - X^{\gamma}, \qquad \gamma \sim 3, \tag{1.16}$$

with X yet another random number uniformly distributed in [0, 1], has approximately this property. On the other hand, they found for OB stars the smaller fraction 21%, which corresponds to $\gamma \sim 1.2$ instead of 3.

Mazeh *et al.* (1992) analysed more closely the Q-distribution for the 23 short-period ($P < 3000$ days) SB2 and SB1 members of the Duquennoy/Mayor sample, using a maximum-likelihood algorithm. They found a mild concentration to equal masses, with about 60% of systems having $0.5 < Q < 1$, which corresponds to $\gamma \sim 1.4$ in the distribution (1.16). They also suggested that this is significantly different from the distribution in wider orbits still, which favours more extreme mass ratios.

Tokovinin (1992) looked at spectroscopic orbital data for $\sim$200 G9–M3 dwarfs, out of which 13 were SB2 and 9 SB1 with $P \leq 3000$ days. Using a maximum-likelihood method, he found a bimodal distribution of secondary masses: 10% of his systems have secondaries in the range 0.32–0.64 $M_{\odot}$, 3% in the range 0.08–0.16 $M_{\odot}$, and the remainder have no (spectroscopic) secondaries. The first peak more-or-less corresponds to $Q \sim 0.5$–1, suggesting that

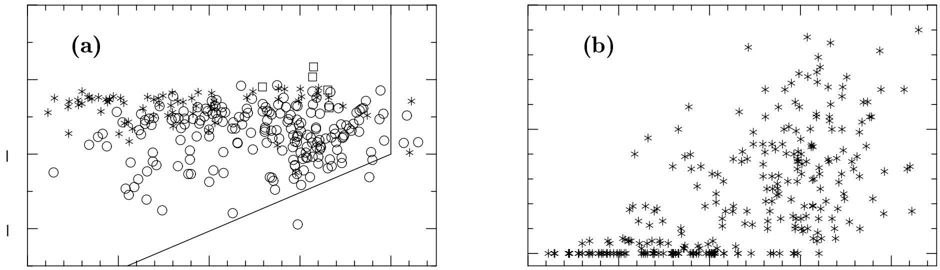

Figure 1.9 The distribution of (a) mass-function and (b) eccentricity against period for 268 G/K giants. In (a), SB1s are shown as circles and SB2s as asterisks. Five triples, the companion being itself a close binary, are shown as squares. Unrecognised triple companions may cause some of the other large values. The sloping line corresponds to a radial velocity amplitude of $K_1 \sim 2$ km/s, roughly the limit of observational detection; the vertical line at $P = 10^4$ days represents an empirical upper limit to period, since longer periods are hard to determine. In (b), all systems are represented by asterisks. The sloping upper boundary and the modest concentration at $e = 0$ probably reflect evolutionary effects rather than primordial properties.

there is still a preference for near-equal masses at moderately short periods. However, the numbers are too few to say if there is a significant departure from distribution (1.16).

The O-star sample of Mason *et al.* (1998) gave a rather different picture from the (much smaller) OB sample of Lucy and Ricco (1979), with about as many systems in the range $Q = 0.4$–0.6 as in the range 0.6–1. In the distribution (1.16) this corresponds to $\gamma \sim 0.8$.

We should emphasise that distributions like (1.16), with just one free parameter (the exponent), can easily be made to fit any statement of the character that a certain percentage of SB2s has $Q > Q_1$, and the remainder have $Q < Q_1$. However it then implies an extrapolation to the smallest mass ratios that may not be warranted, but is very hard to check.

A study of B-type binaries by van Rensbergen (2001), using the catalogue by Batten *et al.* (1989), found distributions slightly biased towards low Q, as in the more complete sample of O stars from Mason *et al.* (1998). His distributions are roughly equivalent to $\gamma \sim 0.8$ and 0.7 for late B and early B systems respectively.

The distinction between distance-limited and magnitude-limited samples is particularly important for mass ratios, since two equal-mass stars can be expected to be twice as bright, and therefore visible over 2.83 times the volume, as systems with a small mass ratio. But for O stars, distances are so great as to be very uncertain. We probably do best to establish the distribution iteratively, using a magnitude-limited sample and then making due allowance for the over-representation of equal-masses down to a given magnitude limit.

Even with data only (or mainly) on single-lined binaries, i.e. a determination only of the mass function and not of the mass ratio, some information can be gleaned about the distribution of mass ratios. Figure 1.9a shows the distribution of measured mass functions – Eq. (1.1) – from a compilation of published data for 268 red giant SBs (G/K, II–IV). SB1s are shown by circles and SB2s by asterisks, but in SB2s only the mass function of the giant is plotted (or the more massive giant, in a few cases where both components are giants). By SB2 in this context we mean stars in which two spectra are seen, even though in many