The Physics of Synchrotron Radiation

ALBERT HOFMANN

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20

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THE PHYSICS OF SYNCHROTRON RADIATION

This book explains the underlying physics of synchrotron radiation and derives its main properties. It is divided into four parts. The first covers the general case of the electromagnetic fields created by an accelerated relativistic charge. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, deriving its distribution in angle, frequency, and polarization modes. The third part looks at undulator radiation. Starting from the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation, the author then discusses strong undulators, emitting more complicated radiation and containing higher harmonics. More general undulators are also considered, with a non-planar (helical) electron trajectory or non-harmonic field. The final part deals with applications and investigates the optics of synchrotron radiation dominated by diffraction due to the small opening angle. It also includes a description of electron-storage rings as radiation sources and the effect of the emitted radiation on the electron beam.

This book provides a valuable reference for scientists and engineers in the field of accelerators, and for all users of synchrotron radiation.

ALBERT HOFMANN received his doctorate in physics from the ETH (Swiss Federal Institute of Technology) in Zürich in 1964. From 1966 to 1972 he was a Research Fellow at the Cambridge Electron Accelerator, a joint laboratory of Harvard University and MIT. He then spent the next ten years working as Senior Physicist at CERN, Geneva. In 1983 he became a professor at Stanford University, working on the Stanford Linear Collider (SLC) and on optimizing the storage rings SPEAR and PEP for synchrotron-radiation use. He spent two years as head of the SLAC beam-dynamics group. He then returned to CERN, in 1987, and was jointly responsible for the commissioning of the Large Electron–Positron ring (LEP). After its completion, he worked on accelerator-physics problems with this machine until his retirement from CERN in 1998.

Over the years Albert Hofmann has done consulting work for other machines, such as the European Synchrotron Radiation Facility (ESRF), the Synchrotron Radiation Research Center (SRRC) in Taiwan, and the Swiss Light Source (SLS). He has taught in over 25 short-term schools on accelerator physics and synchrotron radiation, and has published numerous papers. In 1992 he was elected to become a fellow of the American Physical Society, and in 1996 he received the Robert Wilson Prize from this Society. In 2001 he obtained the degree Doctor honoris causa from the University of Geneva.

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To my wife Elisabeth
for her support

Contents

Preface

Under the rubric of synchrotron radiation we understand the electromagnetic waves emitted by a charge moving with relativistic velocity and undergoing a transverse acceleration. It is characterized by a small opening angle and a high frequency caused by the velocity of the charge being close to that of light. Owing to the relatively simple motion of the charge, the radiation has clear polarization properties. Ordinary synchrotron radiation is emitted by a charge moving on a circular arc determined by a deflecting magnetic field. It has a broad spectrum, a typical frequency being γ^3 times higher than the Larmor frequency of the charge. This spectrum can be modified by varying the curvature of the trajectory $1/\rho$ within a distance smaller than the formation length of the radiation, as is realized in undulators.

Synchrotron radiation has been investigated theoretically for over a century and experimentally for about half this time. Thanks to its unique properties, this radiation has become a research tool for many fields of science and electron-storage rings serving as radiation sources are spread over the whole globe.

This book tries to explain synchrotron radiation from basic principles and to derive its main properties. It is divided into four parts. First the general case of the electromagnetic fields created by an accelerated relativistic charge is investigated. This gives the angular distribution with the small opening angle of the emitted radiation and distinguishes between the 'near' (Coulomb) and the 'far' (radiation) field. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, which we usually call synchrotron radiation. Its distributions in angle, frequency, and polarization modes are derived. Undulator radiation is treated in the next part. We start with the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation. A strong undulator emits radiation that is more complicated and contains higher harmonics. There are more general undulators having a non-planar (helical) electron trajectory or a non-harmonic field. The last part deals with applications and investigates first the optics of synchrotron radiation, which is dominated by diffraction due to the small opening angle. This is followed by a description of electron-storage rings serving as radiation sources and the effect of the emitted radiation on the electron beam.

There are some technical remarks to be made. Throughout the book MKSA units are used. With very few exceptions the radiation field refers to a single positive elementary charge e as a source. For convenience sometimes the radiation emitted by a current I is also given and, in the last chapter, the temporal coherence of the radiation from different particles is considered. As a basis for the properties of the radiation we give first the total emitted power or energy. In the case of ordinary synchrotron radiation we denote by P_{s} the power radiated by the electron *while* it is going through the magnet and by U_{s} the energy radiated during one revolution. For undulators we denote by P_{u} the power radiated in the undulator but averaged over one period and by U_{s} the energy emitted during one traversal through the undulator. These powers and energies can also be expressed in terms of the photon number or photon flux. Distributions in terms of angle and frequency are then given with these total values as a factor that makes it easy to express them in terms of power, energy, photon-number or photon-flux distributions or in other units. Vectors are printed in bold. They are also written as an array with three components between square brackets, like $\mathbf{E} = [E_x,\ E_y,\ E_z]$. For radiation fields the z-component can often be neglected. The remaining two-component vector is written as $E_\perp = [E_x,\ E_y]$. These field components give the polarization of the radiated power. To mark the contributions of the horizontal or vertical polarization to the power, which is of course a scalar, we write it as a sum $P = P_\sigma + P_\pi$. The calculation of synchrotron radiation leads to some integrals that can be expressed in terms of modified Bessel functions or Airy functions. Here the second type is chosen, but the important results are given in both. Some properties, integrals, and sums of Airy and Bessel functions are given in the appendices, partly for convenience and partly because they are not so easy to find. However, this is not meant to provide rigorous mathematical derivations but rather to provide some insight into how some results are obtained.

There are lots of publications on synchrotron radiation and related topics. Apart from well-known books and journals they appear often in laboratory reports and proceedings of workshops. The bibliography to this volume is by no means complete and refers mostly to the topics covered and the methods used to investigate them.

Acknowledgments

I received much help from many people while writing this book. I owe many thanks to my colleague and friend Bruno Zotter from CERN. He not only answered many questions concerning the mathematics I had to use, but also read the whole book and made many suggestions, corrections, and significant improvements. Jim Murphy from Brookhaven National Laboratory read and corrected part of the book and clarified questions concerning mainly coherent radiation. On many occasions I sought advice from my friend and colleague Hermann Winick from Stanford University. Thanks to his insight and experience, he could answer many questions on synchrotron radiation and explained difficult topics to me. I also profited from discussions with many experts in the field: R. Coisson, K. J. Kim, B. M. Kincaid, S. Krinsky, F. Méot, M. Sands, and H. Wiedemann. I also thank the staffs of the laboratories where I had the opportunity to work on and learn about synchrotron radiation: CEA, Cambridge Electron Accelerator, Harvard University – MIT, Cambridge, MA, U.S.A.; CERN, European Laboratory of Particle Physics, Geneva, Switzerland; SLAC, Stanford Linear Accelerator Center, SSRL division, Stanford, CA, U.S.A.; and LNLS, Laboratório Nacional de Luz Síncrotron, Campinas, Brazil.

Notation

$\mathbf{A}$	vector potential
$\mathrm{Ai}(x),\ \mathrm{Ai}'(x)$	Airy function and its derivative
$\mathbf{B}$	magnetic-field vector
$\tilde{\mathbf{B}}(\omega)$	Fourier transformed B-field of radiation
B_0	amplitude of magnetic undulator field
$\tilde{B}_y(k_\mathrm{g})$	weak-magnet Fourier component at k_g
c	speed of light
C_q	quantum excitation factor
$D_x,\ D'_x$	particle-beam-optics dispersion
e	elementary charge
$\mathbf{E}$	electric-field vector
$\tilde{\mathbf{E}}(\omega)$	Fourier-transformed E-field of radiation
$E_\mathrm{e} = m_0c^2\gamma$	particle energy
$E_\gamma = \hbar\omega$	photon energy
$F_\mathrm{s}(\psi, \omega/\omega_c)$	normalized angular spectral density of SR
$F_\mathrm{u}(\theta, \phi)$	normalized angular power density of UR
$h,\ \hbar = h/(2\pi)$	Planck's constant
$\mathcal{H}$	emittance function
$I_{\mathrm{s}2},\ I_{\mathrm{s}3},\ I_{\mathrm{s}4},\ I_{\mathrm{s}5}$	synchrotron-radiation integrals
$J_n(x)$	Bessel function of order n
$J_\epsilon,\ J_x,\ J_y$	longitudinal and transverse damping partitions
$K_{1/3},\ K_{2/3}$	modified Bessel function of order $1/3$, $2/3$
K_f	quadrupole focusing parameter
k_g	wave number of general weak magnet
$k_\mathrm{u} = 2\pi/\lambda_\mathrm{u}$	undulator period wave number
$K_\mathrm{u} = eB_0/(m_0ck_\mathrm{u})$	undulator parameter
K_u^*	reduced plane undulator parameter
K_uh^*	reduced helical undulator parameter
$L_\mathrm{u} = N_\mathrm{u}\lambda_\mathrm{u}$	undulator length

m_0	rest mass of a particle
$\mathbf{n} = \mathbf{r}/r$	unit vector in **r**-direction
$n_B = -\rho^2 K_\mathrm{f}$	field index
$n_\mathrm{s},\ \dot{n}_\mathrm{s}$	photons per revolution, photon flux
$n_\mathrm{u},\ \dot{n}_\mathrm{u}$	photon number per traversal, photon flux
N_u	undulator period number
P_s	instantaneous radiated power of SR
P_u	period-averaged total power of plane UR
$\mathbf{r}(t')$	distance from source to observer
$\mathbf{R}(t')$	vector from origin to particle
$\mathcal{R}$, Φ	polar coordinates in image plane
$r_0 = e^2/(4\pi\epsilon_0 m_0 c^2)$	classical electron radius
$\mathbf{r}_\mathrm{p}$	vector from origin to observer
$\mathbf{S} = [\mathbf{E} \times \mathbf{B}]/\mu_0$	Poynting vector
$S_\mathrm{hm}(\omega_\mathrm{m})$	normalized spectral power of helical UR
$S_\mathrm{s}(\omega/\omega_c)$	normalized spectral power densities of SR
$t = t' + r/c$	observation time
$t_\mathrm{p} = t - r_\mathrm{p}/c$	reduced observation time
$T_0 = 2\pi/\omega_0$	revolution time without straight sections
$T_\mathrm{rev} = 2\pi/\omega_\mathrm{rev}$	revolution time with straight sections
t'	emission time
U_s	energy radiated per turn of SR
$\mathbf{v}(t') = \mathrm{d}\mathbf{R}/\mathrm{d}t'$	particle velocity
V	scalar potential
$\hat{V}$	peak voltage of RF system
w	transverse coordinate, x or y
$X,\ Y$	rectangular coordinates in image plane
α_c	momentum compaction
$\alpha_\epsilon,\ \alpha_\mathrm{h},\ \alpha_\mathrm{v}$	longitudinal and transverse damping rates
$\alpha_\mathrm{f} = e^2/(2\epsilon_0 ch)$	fine structure constant
$\alpha_w = -\beta_w'/2$	particle-optics functions
$\beta = v/c$	normalized velocity
$\boldsymbol{\beta} = \mathbf{v}/c$	normalized velocity vector
β_w	particle-optics functions
β^*	normalized drift velocity in plane undulator
β_h^*	normalized drift velocity in helical undulators
$\gamma = 1/\sqrt{1-\beta^2}$	Lorentz factor
γ_w	particle-optics functions
γ^*	Lorentz factor of drift velocity
γ_h^*	drift Lorentz factor in helical undulators
ϵ_0	vacuum permittivity
$\epsilon_x,\ \epsilon_y$	horizontal and vertical particle-beam emittance

$\epsilon_{\gamma x},\ \epsilon_{\gamma y}$	horizontal and vertical photon-beam emittance
$\boldsymbol{\eta}_x,\ \boldsymbol{\eta}_y$	unit vectors in x- and y-directions
$\lambda_{\text{Comp}} = h/(m_0 c)$	Compton wavelength
λ_{u}	undulator period length
μ_0	vacuum permeability
ρ	bending radius
$\sigma_x,\ \sigma_x'$	RMS electron-beam size and angular spread
φ_{B}	bending angle in a dipole magnet
$\varphi_w(s)$	betatron phase within one turn
φ_{s}	synchrotron phase angle in RF acceleration
ψ	angle between median plane and $\mathbf{r}_{\text{p}}$
$\omega_0 = \beta c/\rho = 2\pi/T_0$	angular velocity, Larmor frequency
ω_1	fundamental UR frequency off axis
ω_{10}	fundamental UR frequency on axis
$\omega_c = 3\omega_0\gamma^3/2$	critical frequency
ω_m	mth harmonic UR frequency off axis
ω_{m0}	mth harmonic UR frequency on axis
$\omega_{\text{rev}} = 2\pi/T_{\text{rev}}$	revolution frequency with straight sections
$\Omega_{\text{u}} = \beta c k_{\text{u}}$	particle-motion frequency in undulator
$\mathrm{d}P/\mathrm{d}\Omega$	power radiated per unit solid angle
$\mathrm{d}^2 P/(\mathrm{d}\Omega\,\mathrm{d}\omega)$	angular spectral radiated power density
$(\ )_\sigma,\ (\ )_\pi$	horizontal, vertical linear polarization
$(\ )_+,\ (\ )_-$	positive, negative helicity circular polarization
$\{\quad\}_{\text{ret}}$	parenthesis evaluated at emission time t'

Part I

Introduction

1

A qualitative treatment of synchrotron radiation

1.1 Introduction

We consider the radiation emitted by a charged particle moving with constant, relativistic velocity on a circular arc. It is called *synchrotron radiation*, or sometimes also *ordinary synchrotron radiation*, abbreviated as SR, to distinguish it from the related case of undulator radiation, abbreviated as UR. We start with a qualitative discussion of synchrotron radiation in order to obtain some insight into its physical properties such as the opening angle, spectrum, and polarization. This will also help us to judge the validity of some approximations used in later calculations.

The physical properties of synchrotron radiation have their basis in the fact that the charge moves with relativistic velocity towards the observer. The charge and the emitted radiation travel with comparable velocities in about the same direction. The fields created by the charge over a relatively long time are received by the observer within a much shorter time interval. This time compression determines the spectrum of synchrotron radiation.

1.2 The opening angle

We consider a charge moving in the laboratory frame F on a circular trajectory with radius of curvature ρ, Fig. 1.1. We go into a frame F′ that moves with a constant velocity that is the same as that of the charge at the instant it traverses the origin. The particle trajectory has in this frame the form of a cycloid with a cusp at the origin. At this location the particle is momentarily at rest, but undergoes an acceleration in the $-x'$-direction. Like any accelerated charge, it emits radiation having an approximately uniform distribution in this frame F′.

We go back to the laboratory frame F by applying a Lorentz transformation. The emitted radiation is now peaked in the forward direction. A photon emitted along the x'-axis in the moving frame F′ appears in the laboratory frame at an angle θ given by

$$\sin\theta = \frac{1}{\gamma} \quad \text{or} \quad \theta \approx \frac{1}{\gamma}, \tag{1.1}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor and $\beta = v/c$ is the normalized velocity. The typical opening angle of the emitted synchrotron radiation is therefore expected to be of

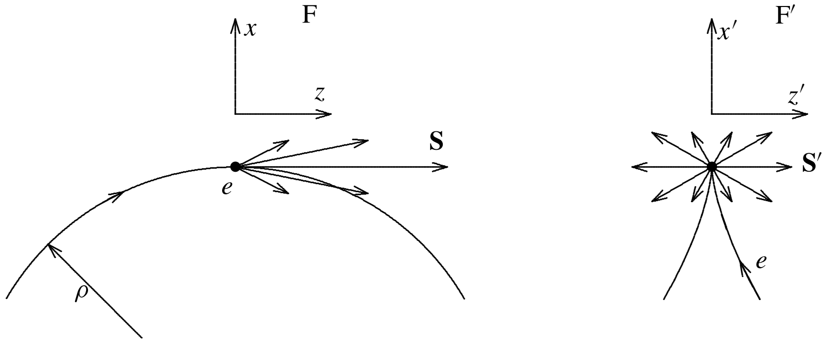

Fig. 1.1. The opening angle of synchrotron radiation.

order $1/\gamma$. For ultra-relativistic particles, $\gamma \gg 1$, the radiation is confined to very small opening angles around the direction of the particle velocity.

1.3 The spectrum emitted in a long magnet

Next we estimate the typical frequency of the emitted radiation. We consider a charge moving on a circular trajectory through a long magnet as shown in Fig. 1.2. We try to estimate the length Δt of the radiation pulse received by the observer P. Owing to the small natural opening angle the observer receives only radiation that was emitted along an arc of approximate angle $\pm 1/\gamma$. Therefore, the radiation observed first was emitted at point A, where the trajectory has an angle $1/\gamma$ with respect to this direction pointing towards the observer, whereas the radiation observed last was emitted at point A′, where the trajectory has a corresponding angle $-1/\gamma$. The length of the radiation pulse seen by the observer is therefore just the difference in travel time between the charge and the radiation for going from point A to point A′:

$$\Delta t = t_{\mathrm{e}} - t_{\gamma} = \frac{2\rho}{\beta\gamma c} - \frac{2\rho \sin(1/\gamma)}{c}.$$

For the ultra-relativistic velocities considered here we have $1/\gamma \ll 1$ and the trigonometric function can be expanded to give

$$\Delta t \approx \frac{2\rho}{\beta\gamma c}\left(1 - \beta + \frac{\beta}{6\gamma^2}\right) \approx \frac{\rho}{\gamma c}\left(\frac{1}{\gamma^2} + \frac{1}{3\gamma^2}\right) = \frac{4\rho}{3c\gamma^3}.$$

Here we use the ultra-relativistic approximation

$$1 - \beta = \frac{1-\beta^2}{1+\beta} \approx \frac{1}{2\gamma^2}. \tag{1.2}$$

From the length Δt of the radiation pulse we get the typical frequency of the spectrum,

$$\omega_{\mathrm{typ}} \approx \frac{1}{\Delta t} \approx \frac{3c\gamma^3}{4\rho}. \tag{1.3}$$

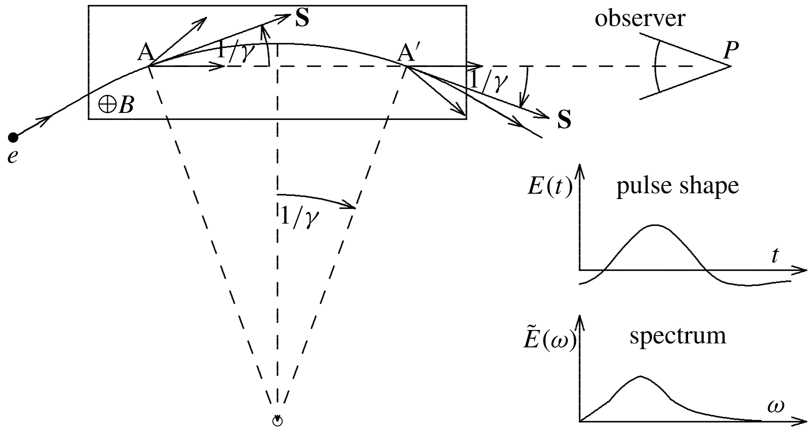

Fig. 1.2. The typical frequency of the synchrotron-radiation spectrum.

Later, on the basis of a quantitative treatment, we will introduce the critical frequency, which is twice as large, $\omega_c = 2\omega_{typ}$. For a large value of the Lorentz factor γ the radiation pulse can become very short and the resulting typical frequency very high.

The above derivation of the typical frequency is quite simple but illustrates some of the most important physical principles of synchrotron radiation. The length of the radiation pulse received is given by the difference in travel time between the particle and the photon for going from point A to point A′. The observed radiation originates from a trajectory arc of approximate length $\ell_r \approx 2\rho/\gamma$. The length L of the magnet has to be larger than this for the above treatment to be valid.

1.4 The spectrum emitted in a short weak magnet

We consider a short weak magnet as shown in Fig. 1.3 with length $L < \rho/\gamma$. It deflects the particle by an angle

$$\Delta\phi = 2\arcsin\left(\frac{L}{2\rho}\right) \approx \frac{L}{\rho} < \frac{1}{\gamma},$$

which is less then the natural opening angle of the radiation. The length Δt_{sm} of the radiation pulse now becomes

$$\Delta t_{sm} = t_e - t_\gamma = \frac{2\rho}{\beta c}\arcsin\left(\frac{L}{2\rho}\right) - \frac{L}{c} \approx \frac{L}{\beta c}(1-\beta) \approx \frac{L}{2c\gamma^2},$$

assuming again that we have the ultra-relativistic case $\beta \approx 1$. The length of the radiation pulse is now proportional to the magnet length L. Reducing it will therefore lead to shorter wavelengths.

The spectrum of the emitted radiation is also modified if the magnetic field changes within the length L, which is the case for undulators. In order for synchrotron radiation to

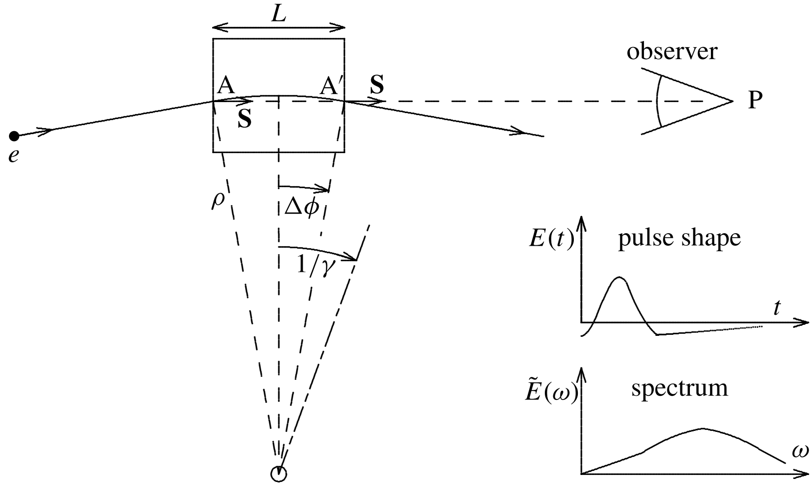

Fig. 1.3. The spectrum radiated in a short magnet.

have a spectrum described by (1.3) it has to be emitted from a magnet with a field that is homogeneous over at least a length of $L > 2\rho/\gamma$. By 'synchrotron radiation' we usually mean the radiation from a long magnet. Sometimes it is also called 'ordinary' synchrotron radiation or 'long-magnet' radiation and will sometimes be abbreviated here to 'SR'. This distinguishes it from undulator or 'short-magnet' radiation. This term 'short magnet' is now commonly used but describes a magnet that is short and weak such that the trajectory angle is everywhere smaller than $1/\gamma$ with respect to the main direction.

1.5 The wave front of synchrotron radiation

In estimating the typical frequency of synchrotron radiation we found that the field which is received by the observer P at the time t within a very short time interval Δt has been emitted by the particle at a different location and at a time t' over a longer time interval $\Delta t'$. Let us consider a particle moving in the general direction towards an observer with a speed close to that of light, emitting pulses of radiation at regular intervals along its trajectory. These pulses are received by the observer at time intervals that are much shorter. The compression of the time sequences Δt of reception compared with the time sequences $\Delta t'$ of emission is stronger the closer the particle velocity is to that of light and the closer its direction to that pointing towards the observer. This is well known from the Doppler effect.

We illustrate this situation in Fig. 1.4 for a charged particle moving with a constant speed $v = \beta c$ ($\beta = 0.8$) anti-clockwise on a circle of radius ρ and emitting a pulse of radiation at regular intervals indicated by small full circles (bullets). These pulses of radiation propagate at the speed of light on circular wave fronts around the sources in their centers. At a certain time t they have reached the situation shown in Fig. 1.4. The pulse emitted first at the time

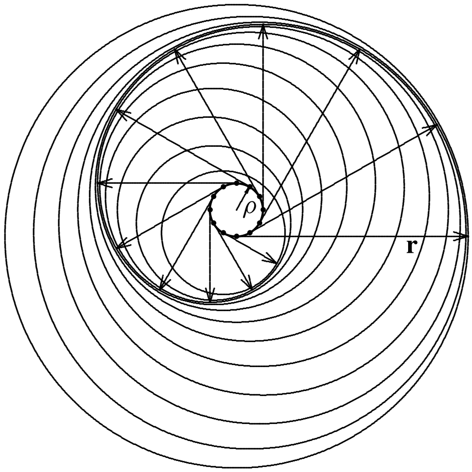

Fig. 1.4. Global propagation of synchrotron radiation for $\beta = 0.9$.

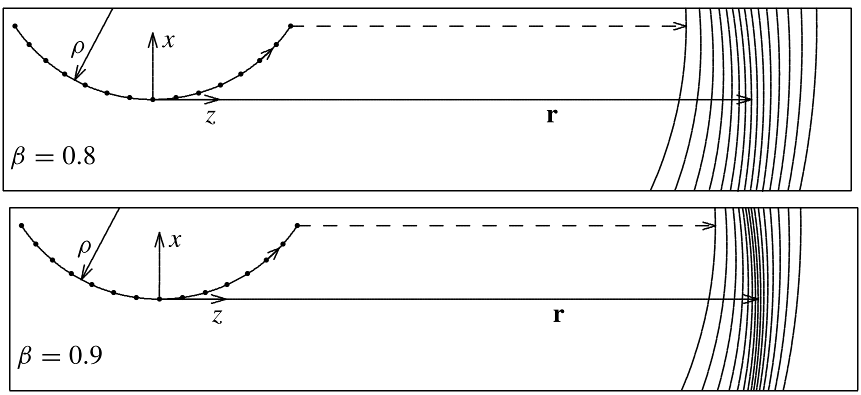

Fig. 1.5. Forward propagation of synchrotron radiation for $\beta = 0.8$ and 0.9.

$t' = 0$ originates from the bottom point and has reached the largest circle. The particle takes some time $\Delta t'$ to reach the next point of emission. Since it moves slower than light the wave emitted at this second point can never catch up with the first one but lags behind only by a small amount in the forward direction indicated by the arrow. Figure 1.4 shows the wave fronts emitted during one revolution of the particle executed at an earlier time. At a certain distance in the forward direction these wave fronts are concentrated in the radial direction. As a consequence an observer at this location receives within a short time interval Δt the radiation emitted during a large interval $\Delta t'$ of the particle motion. In Fig. 1.5 this is illustrated in more detail for the radiation emitted from a finite arc of the trajectory for two velocities $v = \beta c$ of the particle. Clearly the higher velocity ($\beta = 0.9$) leads to a stronger concentration of the wave front than does the lower one ($\beta = 0.8$).

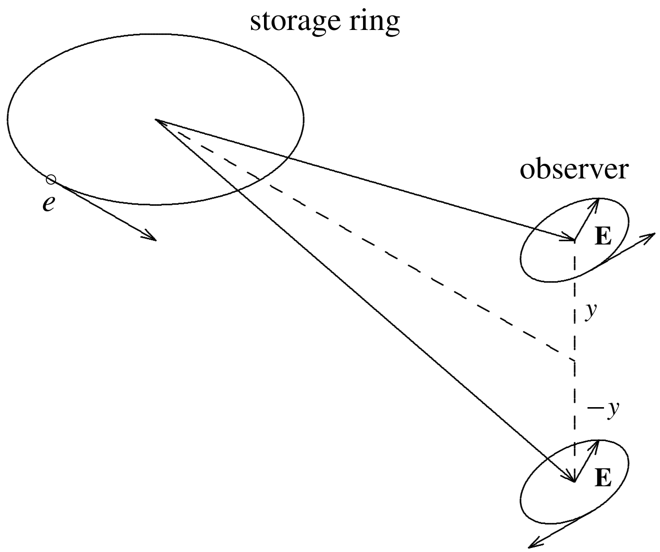

Fig. 1.6. Linear and elliptical polarization of synchrotron radiation.

The emission of short pulses is an artificial picture that we can use in order to obtain a simple illustration. In reality the charge radiates continuously, which is more difficult to draw. Very nice displays of the actual emission of radiation are presented in [1, 2].

We saw at the beginning of this chapter that the radiation is emitted mainly in the forward direction. Therefore, from the wave-front circles drawn in Figs. 1.4 and 1.5 only a limited arc around the forward direction contributes to the field received by the observer.

1.6 The polarization

Since the acceleration of the charge is radial and lies in the plane of the trajectory, we expect that the emitted radiation is mostly linearly polarized, with the electric-field vector also lying in this plane. The radiation observed at a finite angle above or below this plane has some elliptic polarization of opposite helicities, as illustrated in Fig. 1.6.

2

Fields of a moving charge

2.1 Introduction

In the previous chapter we used some qualitative arguments to estimate the basic nature of synchrotron radiation. The results of this exercise are very useful for understanding the underlying physics, estimating the quantities involved, and judging the validity of certain approximations we will make. Now synchrotron radiation is treated in a quantitative manner. We will distinguish between the time t at which the radiation is observed and t' when it was created by the moving charge at a distance r. Since the relation between the two is in general rather complicated, some of the derivations are lengthy. As final results we obtain expressions for the radiation field and the emitted power, which will be applied to calculate synchrotron and undulator radiation in the next two parts. Treatments of synchrotron radiation can be found in many books, journal publications, articles, proceedings of conferences and workshops, and laboratory reports. The first book on the topic of synchrotron radiation [3] was published in 1912. Complete coverage of the topic is presented in [4–8], some of which give also a quantum-mechanical treatment. Many books on electrodynamics treat the radiation from relativistic particles and cover also theoretical aspects of synchrotron radiation [9–13]. On the other hand, many publications on particle accelerators have chapters on synchrotron radiation, giving details of its properties and effects on the electron beam. Among those are the books [14–17]. There are many proceedings from conferences, workshops, and schools and laboratory reports concerned mainly with accelerators but containing also articles on synchrotron radiation [18–20]. Furthermore, there are several handbooks and proceedings concerned mainly with the science done with synchrotron radiation [21–24], which describe the properties and technical possibilities of this source [25]. There are overviews on the history of synchrotron radiation, such as [26], which gives mainly the early development, and [27], which concentrates on the work done in the U.S.S.R.

2.2 The particle motion relevant to the retarded potentials

To relate the observed radiation to the motion of charge and vice versa we invoke so-called retarded potentials and fields, which have their basis in the finite propagation velocity c

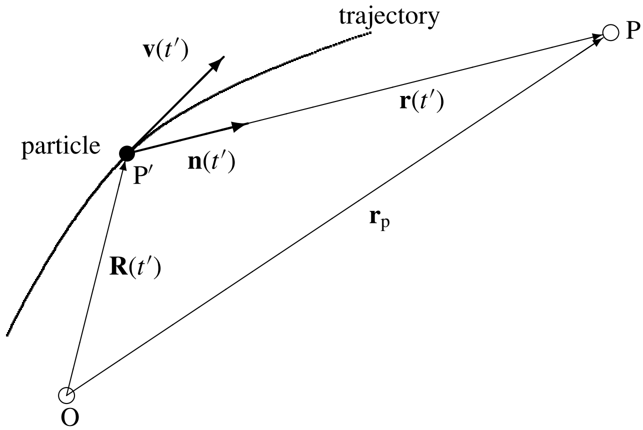

Fig. 2.1. The particle trajectory and radiation geometry.

of the electromagnetic radiation. To calculate the fields measured at time t by a stationary observer we have to know the position and motion of the charge at this earlier time t' of emission.

We discuss now the motion relevant for these two time scales and consider an *elementary positive charge e* moving on a trajectory given by the vector $\mathbf{R}(t')$ and creating an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$. These fields are measured at time t by the observer located at P as illustrated in Fig. 2.1. We introduce a vector $\mathbf{r}$ with absolute value r, pointing from the location P′ of emission to the observer P. Owing to the finite propagation velocity c, the field received at time t by the observer P had to have been emitted by the source P′ at the earlier time t' given by the relation

$$\boxed{t = t' + \frac{r(t')}{c}.} \tag{2.1}$$

Therefore, we have to know the position $\mathbf{R}(t')$ and velocity $\mathbf{v}(t') = \mathrm{d}\mathbf{R}/\mathrm{d}t'$ of the charged particle at this earlier time t'. We have for the vectors $\mathbf{R}$ (pointing from the origin to the radiating charge), $\mathbf{r}_\mathrm{p}$ (pointing from the origin to the observer), and $\mathbf{r}$ (pointing from the charge to the observer) the relation

$$\mathbf{R}(t') + \mathbf{r}(t') = \mathbf{r}_\mathrm{p} = \text{constant}. \tag{2.2}$$

Differentiating this with respect to t' gives the change of the vector $\mathbf{r}$,

$$\frac{\mathrm{d}\mathbf{r}(t')}{\mathrm{d}t'} = -\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}t'} = -\mathbf{v}(t') = -c\boldsymbol{\beta}(t'), \tag{2.3}$$

from which we obtain the corresponding change of its absolute value $r = |\mathbf{r}|$:

$$\mathbf{r}\,\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t'} = \frac{1}{2}\,\frac{\mathrm{d}(\mathbf{r}^2)}{\mathrm{d}t'} = r\,\frac{\mathrm{d}r}{\mathrm{d}t'} = -(\mathbf{r}\cdot\mathbf{v}).$$

Introducing the unit vector

$$\mathbf{n} = \mathbf{r}/r \tag{2.4}$$

pointing from the charge in the direction towards the observer gives for the change of the distance between the source and the observer at time t'

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = -(\mathbf{n} \cdot \mathbf{v}) = -c(\mathbf{n} \cdot \boldsymbol{\beta}), \tag{2.5}$$

which is just the negative particle-velocity component of the particle in the direction towards the observer as shown in Fig. 2.1. The differential relation between the two time scales t' and t is obtained from (2.1):

$$\boxed{\mathrm{d}t = \left(1 + \frac{1}{c}\frac{\mathrm{d}r}{\mathrm{d}t'}\right)\mathrm{d}t' = (1 - \mathbf{n} \cdot \boldsymbol{\beta})\,\mathrm{d}t'.} \tag{2.6}$$

2.3 The retarded electromagnetic potentials

In this section we derive expressions for the electromagnetic potentials $\mathbf{A}(t)$ and $V(t)$ observed at P and created by a charge moving along a trajectory given by the vector $\mathbf{R}(t')$. This result will be used in the next section to obtain the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ which are related to the scalar and vector potentials V and $\mathbf{A}$ through Maxwell's equations, which can be found in standard textbooks on electrodynamics listed earlier:

$$\begin{aligned} \mathbf{E} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = \text{grad } V - \frac{\partial \mathbf{A}}{\partial \mathrm{t}} \\ \mathbf{B} &= [\nabla \times \mathbf{A}] = \text{curl}\,\mathbf{A} \end{aligned} \tag{2.7}$$

with the Lorentz convention $\nabla \cdot \mathbf{A} = -\dot{V}/c^2$. The vector potential $\mathbf{A}$ is measured in units of $\mathrm{V\,s\,m^{-1}}$.

The potentials created by time-dependent charge $\eta(t')$ and current density $\mathbf{J}(t')$ are given by the expressions

$$\begin{aligned} V(t) &= \frac{1}{4\pi\epsilon_0}\int \frac{\eta(t')}{r(t')}\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z' \\ \mathbf{A}(t) &= \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(t')}{r(t')}\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'. \end{aligned}$$

The above expressions are very similar to those used to calculate the potentials of static charge and stationary current distributions. However, here the charges move and the local charge and current densities change. Since the potentials created propagate at the speed of light, the signals received by the observer P depend on the positions of the charges at the earlier time t'. The integration is carried out over the coordinates x', y', and z' of the earlier distribution.