Modeling Monetary Economies

Second Edition

Bruce Champ
Scott Freeman

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The approach of this text for upper-level undergraduates is to teach monetary economics using the classical paradigm of rational agents in a market setting. Too often monetary economics has been taught as a collection of facts about existing institutions for students to memorize. By teaching from first principles instead, the authors aim to instruct students not only in the monetary policies and institutions that exist today in the United States but also in what policies and institutions may or should exist tomorrow and elsewhere. The text builds on a simple, clear monetary model and applies this framework consistently to a wide variety of monetary questions. The authors have added in this second edition new material on speculative attacks on currencies, social security, currency boards, central banking alternatives, the payments system, and the Lucas model of price surprises. Discussions of many topics have been extended, presentations of data greatly expanded, and new exercises added.

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Preface

WE OFFER THIS book as an undergraduate-level exposition of lessons about monetary economics gleaned from overlapping generations models of money. Assembling recent advances in monetary theory for the instruction of undergraduates is not a quixotic goal; these models are well within the reach of undergraduate students at the intermediate and advanced levels. These elegantly simple models strengthen our fundamental understanding of the most basic questions in monetary economics: How does money promote exchange? What should serve as money? What causes inflation? What is the cost of inflation?

This approach to teaching monetary economics follows the profession's general recoginition of the need to build the microeconomic foundations of monetary and other macroeconomic topics – that is, to explain aggregate economic phenomena as the implications of the choices of rational individuals who seek to improve their welfare within their limited means. The use of microeconomic foundations makes macroeconomics easier to understand because the performance of abstract economic processes such as gross national product and inflation is linked to something intuitively understood by all – rational individual behavior. It also brings powerful microeconomic tools familiar to undergraduates, such as indifference curves and budget lines, to bear on the questions of interest. Finally, the joining of micro- and macroeconomics introduces an element of consistency across undergraduate studies. Certainly, students will be puzzled if taught that people are rational and prices clear markets when studied by microeconomists but not when studied by macroeconomists.

Inertia and tradition, however, have mired the teaching of monetary economics in a swamp of institutional details, as if monetary economics were only an unchanging set of facts to be memorized. The rapid pace of change in the financial world belies this view. Undergraduates need a way to analyze a wide variety of monetary events and institutional arrangements because the events and institutions of the future will not be the same as those that the students studied in the classroom. The teaching of analysis, the heart of a liberal education, is best accomplished by having the students analyze clear, explicit, and internally consistent models. In this way, the students may uncover the links between the assumptions underlying the models and the performance of these model economies and thus may apply their lessons to new events or changes in government priorities and policies.

This book implements our goals by starting with the simplest model of money – the basic overlapping generations model – which we analyze for its insights into the most basic questions of monetary economics, including the puzzling demand for useless pieces of paper and the costs of inflation. Of course, such a simple model will not be able to discuss all issues of monetary economics. Therefore, we proceed in successive chapters by asking which features of actual economies the simple model does not address. We then introduce these neglected features into the model to enable us to discuss these more advanced topics. We believe this gradual approach allows us to build, step by step, an integrated model of the monetary economy without overwhelming the students.

The book is organized into three parts of increasing complexity. Part I examines money in isolation. Here we take up the questions of the demand for fiat money, a comparison of fiat and commodity monies, inflation, and exchange rates. In Part II, we add capital, to study money's interaction with other assets, banking, the intermediation of these assets into substitutes for fiat money, and alternative arrangements of central banking. In Part III, we look at the effects of money on saving, investment, and output through its effect on nonmonetary government debt.

This book is written for undergraduate students. Its mathematical requirements are no more advanced than the understanding of basic graphs and algebra; calculus is not required. (Those who want to use calculus can find a basic exposition of this approach in the appendix to Chapter 1.) Although we hope the book may also prove useful to graduate students as a primer in monetary theory, the main text is pitched at the undergraduate level. This pitch has held us back from a few demanding topics, such as nonstationary equilibria, but we hope the reader will be satisfied by the large number of topics we present in simple, clear models within a single basic framework. Material that is difficult but within the grasp of advanced undergraduates is set apart in appendices and thus easily skipped or inserted. The appendices also contain extensions, like the model of credit, that many instructors may want to use but that are not essential to the exposition of the main topics.

The references display the most tension between the needs of undergraduates and the technical base in which this approach originated. Whenever possible we reference related material written for undergraduates or general audiences; these references are marked by asterisks. We also reference the works from which our models and data have been drawn. Finally, where undergraduate-level references were not available, we have inserted references to a few academic articles and surveys to offer graduate and advanced undergraduate students some places to start with more advanced work. These are not intended to be a full survey of the advanced literature.

The choice of topics to be covered was also difficult. We make no claim to encyclopedic coverage of every topic or opinion related to monetary economics. We limited coverage to the topics most directly linked to money, covering banking (but not finance in general) and government debt (but not macroeconomics in general). We insisted on ideas consistent with fully rational people operating in explicitly specified environments to promote the unity and consistency of our approach across topics. We also selected topics tractably teachable in the basic framework of the overlapping generations model. Finally, we offer what we best know and understand. We hope individual instructors will build on our foundations to fill any gaps.

To reduce these gaps, in this second edition we have added new material on speculative attacks, the not-very-monetary topic of social security, currency boards, central banking alternatives, the payments system, and the Lucas model of price surprises. We have greatly expanded our presentations of data and have added new exercises.

Many have contributed to the development of this book. We owe Neil Wallace a tremendous intellectual debt for impressing upon us the importance of microeconomic theory in monetary economics. Many others have provided helpful suggestions, criticisms, encouragement, and other assistance during the writing of this book, including David Andolfatto, Leonardo Auernheimer, Robin Bade, Valerie Bencivenga, Mike Bryan, John Bryant, Douglas Dacy, Silverio Foresi, Christian Gilles, Paul Gomme, Joseph Haslag, Dennis Jansen, David Laidler, Kam Liu, Mike Loewy, Helen O'Keefe, John O'Keefe, Michael Parkin, Dan Peled, Tom Sargent, Pierre Siklos, Bruce Smith, Ken Stewart, Dick Tresch, François Velde, Paula Hernandez Verme, Warren Weber, and Steve Williamson. We thank the large number of students at Boston College, the University of California at Santa Barbara, the University of Western Ontario, Fordham University, and the University of Texas at Austin, who persevered through the various preliminary versions of this book. We are grateful for their patience and suggestions. The views stated in this book are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

Bruce Champ
Scott Freeman

Part I
Money

IN PART I we develop and learn to work with our most basic model of money, applying it to the study of fiat and commodity monies, inflation, and international monetary systems. In studying each of these topics, we will examine how money can facilitate trade and we will ask which form of money or system of exchange does so most efficiently.

Chapter 1 begins with a simple model of money designed to answer the most basic question in monetary economics: Why do people use money? Why do people value pieces of paper that cannot be consumed? With this model, we discover that intrinsically worthless pieces of paper can have value by providing a means by which individuals can acquire goods that they do not possess. Because of this, we also find that the introduction of money into an economy improves welfare. The model of Chapter 1 serves as the foundation on which we later build more complex models.

Chapter 2 considers two alternatives to the paper money of Chapter 1 – barter and commodity money. This chapter first presents a model that demonstrates, in a multiple good model, why barter, the direct trade of goods owned for goods desired, may be more costly than monetary exchange, the trade of goods owned for money and the subsequent trade of money for goods desired. Chapter 2 also demonstrates that commodities may be exchanged as monies. We will ask which commodities make the best monies and we will compare the efficiency of exchange with commodity money to exchange with fiat money.

Chapter 3 uses the basic model of money of Chapter 1 to analyze the effects of the expansion of the money supply on prices, the willingness of people to use money, and the welfare of individuals. We also see how the printing of fiat money can be used to raise revenue for government expenditures. We compare the efficiency of this revenue device with that of taxes.

Chapter 4 examines international monetary systems. We discover some of the important determinants of exchange rates, we discuss floating versus fixed exchange rate regimes and speculative attacks, and we compare alternative international monetary systems.

Chapter 5 looks at some effects of price surprises on labor and output. We introduce the concept of rational expectations and show that a positive correlation between price surprises and output may not imply that the monetary authority is able to increase output in any systematic way. More generally, the chapter is intended to demonstrate the pitfalls of giving policy advice based on purely statistical correlations without understanding the workings of the economy that generated those correlations.

Chapter 1

A Simple Model of Money

Building a Model of Money

IN THIS BOOK we will try to learn about monetary economies through construction of a series of model economies that replicate essential features of actual monetary economies. All such models are simplifications of the complex economic reality in which we live. They may be useful, however, if they are able to illustrate key elements of the behavior of people who choose to hold money and to predict the reactions of important economic variables such as output, prices, government revenue, and public welfare to changes in policies that involve money. We start our analysis with the simplest conceivable model of money. We will learn what we can from this simple model and then ask how the model fails to represent reality adequately. Throughout the book we will try to correct the model's oversights by adding, one by one, the features that it lacks.

To arrive at the simplest possible model of money, we must ask ourselves what features are essential to monetary economies. The demand for money is distinct from the demand for the goods studied elsewhere in economics. People want goods for the utility received from their consumption. In contrast, people do not want money in order to consume it; they want money because money helps them get the things they want to consume. In this way, money is a **medium of exchange** – something acquired to make it easier to trade for the goods whose consumption is desired.

A model of this distinction in the demand for money therefore requires two special features. First, there must be some "friction" to trade that inhibits people from directly acquiring the goods they desire in the absence of money. If people could costlessly trade what they have for what they want, there would be no role for money.

Second, someone must be willing to hold money from one period to the next. This is necessary because money is an asset held over some period of time, however short, before it is spent. Therefore, we will look for models in which there is always someone who will live into the next period.

Two possible frameworks meet this second requirement. People (or households) could live infinite lives or could live finite lives in generations that overlap (so that some but not all people will live into the next period). For many of the topics we study, life span does not matter. We identify where it does matter in Appendix B of Chapter 15, where infinitely lived households are studied in detail.

With the exception of that appendix, we concentrate on the second framework – the **overlapping generations model**. This model, introduced by Paul Samuelson (1958), has been applied to the study of a large number of topics in monetary theory and macroeconomic theory. Among its desirable features are the following:

- Overlapping generations models are highly tractable. Although they can be used to analyze quite complex issues, they are relatively easy to use. Many of their predictions may be described on a simple two-dimensional graph.
- Overlapping generations models provide an elegantly parsimonious framework in which to introduce the existence of money. Money in overlapping generations models dramatically facilitates exchange between people who otherwise would be unable to trade.
- Overlapping generations models are **dynamic**. They demonstrate how behavior in the present can be affected by anticipated future events. They stand in marked contrast to **static** models, which assume that only current events affect behavior.

We begin this chapter with a very simple version of an overlapping generations model. As we proceed through the book, we introduce extensions to this basic model. These extensions allow us to analyze a variety of interesting issues. For now, let us turn to the development of the basic overlapping generations model.

The Environment

In the basic overlapping generations model, individuals live for two periods. We call people in the first period of life **young** and those in the second period of life **old**.

The economy begins in period 1. In each period $t \geq 1$, N_t individuals are born. Note that we index time with a subscript. For example, N_2 is our notation for the number of individuals born in period 2. The individuals born in periods $1, 2, 3, \ldots$ are called the **future generations** of the economy. In addition, in period 1 there are N_0 members of the **initial old**.

Hence, in each period t, there are N_t young individuals and N_{t-1} old individuals alive in the economy. For example, in period 1, there are N_0 initial old individuals and N_1 young individuals who were born at the beginning of period 1.

For simplicity, there is only one good in this economy. The good cannot be stored from one period to the next. In this basic setup, each individual receives an **endowment** of the consumption good in the first period of life. The amount of this

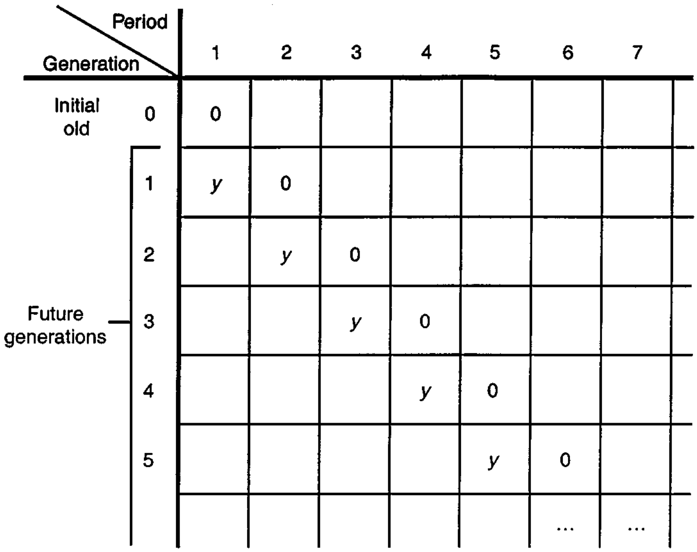

Figure 1.1. The pattern of endowments. In each period t, generation t is born. Each individual lives for two periods. Individuals are endowed with y units of the consumption good when young and 0 units when old. In any given period, one generation of young people and one generation of old people are alive. The name of this model, the overlapping generations model, follows from this generational structure.

endowment is denoted as y. Each individual receives no endowment in the second period of life. This pattern of endowments is illustrated in Figure 1.1.

We can also interpret the endowment as an endowment of labor – the ability to work. By using this labor endowment (by working), the individual is able to obtain a real income of y units of the consumption good.

Preferences

Individuals consume the economy's sole commodity and obtain satisfaction or, in the economist's jargon, utility from having done so.

Future Generations

Members of future generations in an overlapping generations model consume both when young and when old. An individual member's utility therefore depends on the combination of personal consumption when young and when old. We make the following assumptions about an individual's preferences about consumption:

1. For a given amount of consumption in one of the periods, an individual's utility increases with the consumption obtained in the other period.

2. Individuals like to consume some of this good in both periods of life. An individual prefers the consumption of positive amounts of the good in both periods of life over the consumption of any quantity of the good in only one period of life.
3. To receive another unit of consumption tomorrow, an individual is willing to give up more consumption today if the good is currently abundant than if it is scarce relative to consumption tomorrow.

With these assumptions, we are assuming that individuals are capable of ranking combinations (or bundles) of the consumption good over time in order of preference. We denote the amount of the good that is consumed in the first period of life by an individual born in period t with the notation $c_{1,t}$. Similarly, $c_{2,t+1}$ denotes the amount the same individual consumes in the second period of life. It is important to note that $c_{2,t+1}$ is consumption that actually occurs in period $t + 1$, when the person born at time t is old. When the time period is not crucial to the discussion, we denote first- and second-period consumption as c_1 and c_2.

Suppose we offer an individual the following consumption choices:

- Bundle A, which consists of 3 units of the consumption good when young and 6 units of the consumption good when old. We denote this bundle as $c_1 = 3$ and $c_2 = 6$.
- Bundle B, which consists of 5 units of the consumption good when young and 4 units of the consumption good when old ($c_1 = 5$, $c_2 = 4$).

By assuming that an individual can rank these bundles, we are saying that this individual can state a preference for bundle A over bundle B, for bundle B over bundle A, or equal happiness with either bundle. The individual can rank any number of bundles of the consumption good that we might offer in this manner.

It will be extremely useful to portray an individual's preferences graphically. We do this with indifference curves. An **indifference curve** connects all consumption bundles that yield equal utility to the individual. In other words, if offered any two bundles on a given indifference curve, the individual would say, "I do not care which I receive; they are equally satisfying to me." In the preceding example, if the individual were indifferent to bundles A and B, then those two bundles would lie on the same indifference curve. Figure 1.2 displays a typical indifference curve.

On this indifference curve we show the two points A and B from our earlier example. We also illustrate a third point, C, representing the bundle $c_1 = 11$ and $c_2 = 2$. Because C lies on the same indifference curve as points A and B, point C yields the same level of utility as points A and B for the individual. In fact, any point along the illustrated indifference curve represents a bundle that yields the same utility level.

Note some features of the indifference curve. The first feature is that the curve gets flatter as we move from left to right. This is how indifference curves represent assumption 3. This property of indifference curves is called the assumption of **diminishing marginal rate of substitution**. To illustrate this assumption, start

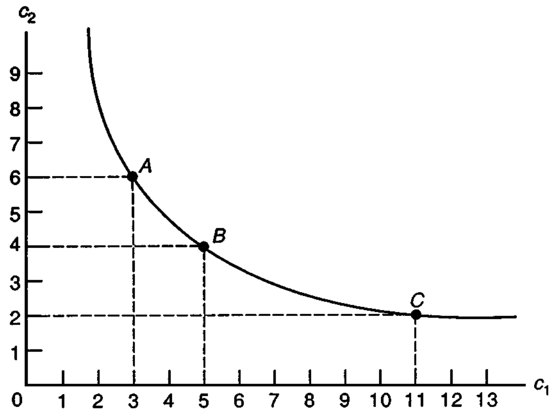

Figure 1.2. An indifference curve. Individual preferences are represented by indifference curves. The figure portrays an indifference curve for a typical individual. Along any particular indifference curve, utility is constant. Here, the individual is indifferent between points A, B, and C.

at point A, where $c_1 = 3$ and $c_2 = 6$. Suppose we reduce the individual's second period consumption by 2 units. The indifference curve tells us that, to keep the individual's utility constant, we must compensate the individual by providing 2 more units of first-period consumption. This places the individual at point B on the indifference curve. Now suppose we reduce second-period consumption by another 2 units. To remain indifferent, 6 more units of first-period consumption must be given to the individual. In other words, we must compensate the individual with ever-increasing amounts of first-period consumption as we successively cut second-period consumption. This should make intuitive sense; individuals are more reluctant to give up something they do not have much of to begin with.

Take food and clothing as an example. A person who has a large amount of clothing and very little food would be willing to give up a fairly large amount of clothing for another unit of food. Conversely, this person would be willing to give up only a small amount of food to obtain another unit of clothing.

This assumption of diminishing marginal rate of substitution is captured by drawing an indifference curve so that it becomes flatter as we move downward and to the right along the curve.

We also assume that the indifference curves become infinitely steep as we approach the vertical axis and perfectly flat as we approach the horizontal axis. The curves never cross either axis. This might be justified by saying that consuming nothing in any one period would mean horrible starvation, to which consuming even a small amount is preferable. This is assumption 2.

It is also important to keep in mind that the indifference curves are dense in the (c_1, c_2) space. This means that if you pick a combination of first- and second-period

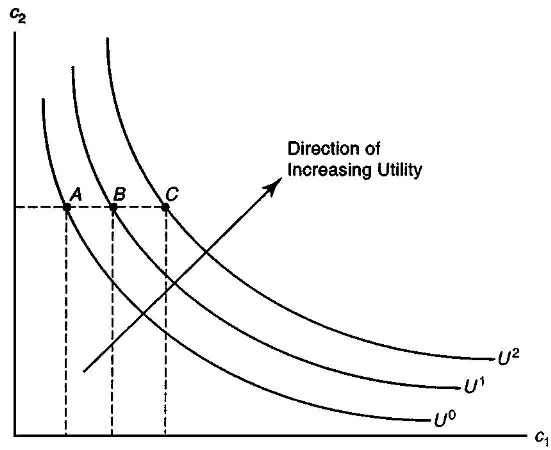

Figure 1.3. An indifference map. An indifference map consists of a collection of indifference curves. For a constant amount of consumption in one period, individuals prefer a greater amount of consumption in the other period. For this reason, individuals prefer point C to point B and point B to point A. Utility increases in the general direction of the arrow.

consumption, there is an indifference curve running through that point. However, to avoid clutter, we normally show only a few of these indifference curves. A group of indifference curves shown on one graph is often called an **indifference map**. Figure 1.3 illustrates an indifference map that obeys our assumptions.

Note that utility is increasing in the direction of the arrow. How do we know this? Compare points A, B, and C. Each of these bundles gives the individual the same amount of second-period consumption. However, moving from point A to B to C, the individual receives more and more first-period consumption. Hence, the individual will prefer point B to point A. Likewise, point C will be preferable to both points A and B. This is assumption 1.

It is often useful to draw an analogy between an indifference map and a contour map that shows elevation. On a contour map, the curves represent points of constant elevation; on an indifference map, the curves represent points of constant utility. Extending the analogy, if we think of traversing the indifference map in a northeasterly direction, we would be going uphill. In other words, utility would be increasing. In fact, an indifference map, like a contour map, is merely a handy way to illustrate a three-dimensional concept on a two-dimensional drawing. The three dimensions here are first-period consumption, second-period consumption, and utility.

One other important concept about our individual's ranking of preferences is that they are transitive. If an individual prefers bundle B to bundle A and bundle C to bundle B, then that individual must also prefer bundle C to bundle A. Graphically, this implies that indifference curves cannot cross. To do so would violate this property of transitivity and assumption 1. To see this, refer to Figure 1.4. In this figure, we have

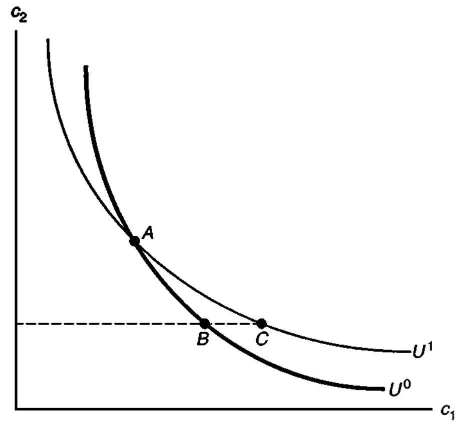

Figure 1.4. Indifference curves cannot cross. By our first assumption about preferences, the individual whose preferences are represented by these indifference curves prefers bundle C over bundle B because bundle C consists of more first-period consumption and the same amount of second-period consumption compared with bundle B. However, because the individual must be indifferent between all three bundles, A, B, and C, a contradiction arises. Our assumptions rule out the possibility of indifference curves that cross.

portrayed two indifference curves that cross at point A. We know that indifference curves represent bundles that give an individual the same level of utility. In other words, the individual whose preferences are represented by Figure 1.4 is indifferent between bundles A and B, because they lie on the same indifference curve U^0. Similarly, the individual must be indifferent between bundles A and C on indifference curve U^1. We see, then, that the individual is indifferent between all three bundles. However, if we compare bundles B and C, we also observe that they consist of the same amount of second-period consumption, but C contains more first-period consumption than B. By assumption 1, the individual must prefer C to B. But this contradicts our earlier statement about indifference between the three bundles. For this reason, indifference curves that cross violate our assumptions about preferences.

The Initial Old

The preferences of the initial old are much easier to describe than those of future generations. The initial old live and consume only in the initial period and thus simply want to maximize their consumption in that period.

The Economic Problem

The problem facing future generations of this economy is very simple. They want to acquire goods they do not have. Each has access to the nonstorable consumption good only when young but wants to consume in both periods of life. They must therefore find a way to acquire consumption in the second period of life and then decide how much they will consume in each period of life.[1]

We will examine, in turn, two solutions to this economic problem. The first, a centralized solution, proposes that an all-knowing, benevolent planner will allocate the economy's resources between consumption by the young and by the old. In the second, decentralized solution, we allow individuals to use money to trade for what they want. We will then compare the two solutions and ask which is more likely to offer individuals the highest utility. The answer will help to provide a first illustration of the economic usefulness of money.

Feasible Allocations

Imagine for a moment that we are central planners with complete knowledge of and total control over the economy. Our job is to allocate the available goods among the young and old people alive in the economy at each point in time.

As central planners, under what constraint would we operate? Put simply, it is that at any given time we cannot allocate more goods than are available in the economy. Recall that only the young people are endowed with the consumption good at time t. There are N_t of these young people at time t. We have

$$(\text{total amount of consumption good})_t = N_t y. \tag{1.1}$$

Suppose that every member of generation t is given the same lifetime allocation $(c_{1,t}, c_{2,t+1})$ of the consumption good (our society's view of equity). In this case, total consumption by the young people in period t is

$$(\text{total young consumption})_t = N_t c_{1,t}. \tag{1.2}$$

Furthermore, total old consumption in period t is

$$(\text{total old consumption})_t = N_{t-1} c_{2,t}. \tag{1.3}$$

Let us make sure the notation is clear. Recall that the old people at time t are those who were born at time $t-1$. There were N_{t-1} of these people born at time $t-1$. Furthermore, recall that $c_{2,t}$ denotes the second period (time t) consumption by someone who was born at time $t-1$. This implies that total consumption by the old at time t must be $N_{t-1} c_{2,t}$.

[1] We could make this model more complex by assuming that there are many types of goods and many periods in which to consume them. A model with many types of goods is introduced in Chapter 2. We will see, however, that this simple model is all that is needed to illustrate a demand for money.

Total consumption by young and old is the sum of the amounts in Equations 1.2 and 1.3. We are now ready to state the constraint facing us as central planners: total consumption by young and old cannot exceed the total amount of available goods (Equation 1.1) . In other words,

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y. \tag{1.4}$$

For simplicity, we assume for now that the population is constant ($N_t = N$ for all t). In this case, we rewrite Equation 1.4 as

$$N c_{1,t} + N c_{2,t} \leq N y.$$

Dividing through by N, we obtain the per capita form of the constraint facing us as central planners:

$$c_{1,t} + c_{2,t} \leq y. \tag{1.5}$$

For now, we are also concerned with a stationary allocation. A **stationary allocation** is one that gives the members of every generation the same lifetime consumption pattern. In other words, in a stationary allocation, $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for every period $t = 1, 2, 3, \ldots$. However, it is important to realize that a stationary allocation does not necessarily imply that $c_1 = c_2$. With a stationary allocation, the per capita constraint becomes

$$c_1 + c_2 \leq y. \tag{1.6}$$

This represents a very simple linear equation in c_1 and c_2, which is illustrated in Figure 1.5.

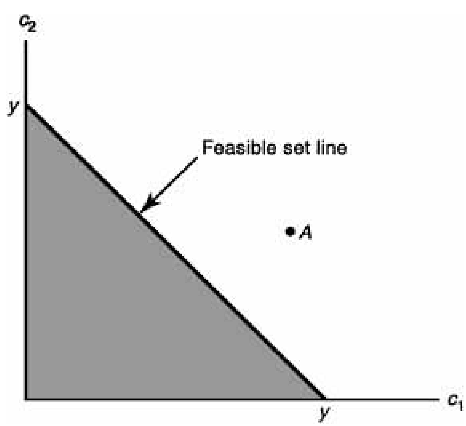

Figure 1.5. The feasible set. The feasible set, the gray triangle, represents the set of possible allocations that can be attained given the resources available in the economy. Points outside the feasible set, such as point A, are unattainable given the resources of the economy.

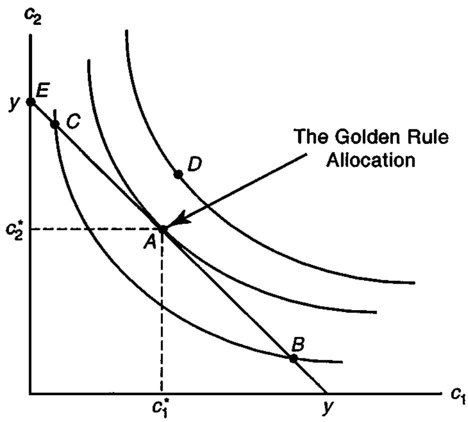

Figure 1.6. The golden rule allocation. The golden rule allocation is the stationary, feasible allocation of consumption that maximizes the welfare of future generations. It is located at a point of tangency between the feasible set line and an indifference curve (point A). This is the highest indifference curve in contact with the feasible set. As drawn, the golden rule allocation A allocates more goods to people when old than when young ($c_2^* > c_1^*$), but this is arbitrary. The tangency can just as easily have been drawn at a point where $c_2^* < c_1^*$.

The set of stationary, feasible, per capita allocations – the feasible set, for short – is bounded by the triangle in the diagram. We refer to the triangular region as the **feasible set**. The thick diagonal line on the boundary of the feasible set is called the **feasible set line**. The feasible set line represents Equation 1.6, evaluated at equality.

The Golden Rule Allocation

If we now superimpose a typical individual's indifference map on this diagram, we can identify the preferences of future generations among feasible stationary allocations. This is done in Figure 1.6.

The feasible allocation that a central planner selects depends on the objective. One reasonable and benevolent objective is the maximization of the utility of the future generations, an objective we call the **golden rule**. The golden rule in Figure 1.6 is represented by point A, which offers each individual the consumption bundle (c_1^*, c_2^*). This combination of c_1 and c_2 yields the highest feasible level of utility over an individual's entire lifetime. Note that the golden rule occurs at the unique point of tangency between the feasible set boundary and an indifference curve. Any other point that lies within the feasible set yields a lower level of utility. For example, points B and C are feasible because they lie on the boundary of the feasible set. However, they lie on an indifference curve that represents a lower level of utility than the one on which point A lies. Point D is preferable to point A, but it is unattainable. The endowments of the economy simply are not large enough to support the allocation implied by point D.

The Initial Old

It is important to consider the welfare of all participants in the economy – including the initial old – when considering the effects of any policy. Although the golden rule allocation maximizes the utility of future generations, it does not maximize the utility of the initial old. To see this, recall that the initial old's utility depends solely and directly on the amount of the good they consume in their second period of life. The goal of the initial old is to get as much consumption as possible in period 1, the only period in which they live. (You may want to imagine that the initial old also lived in period 0; however, because this period is in the past, it cannot be altered by the central planner, who assumes control of the economy in period 1.) If the central planner's goal were to maximize the welfare of the initial old, the planner would want to give as much of the consumption good as possible to the initial old. This would be accomplished among stationary feasible allocations at point E of Figure 1.6, which allocates y units of the good for consumption when old (including consumption by the initial old) and nothing for consumption when young.

This stationary allocation, which implies that people consume nothing when young, would not maximize the utility of the future generations. They prefer the more balanced combination of consumption when young and old represented by (c_1^*, c_2^*). Faced with this conflict in the interests of the initial old and the future generations, an economist cannot choose among them on purely objective grounds. Nevertheless, the reader will find that, on subjective grounds (influenced by the fact that there are an infinite number of future generations and only a single generation of initial old), we tend to pay particular attention to the golden rule in this book.

Decentralized Solutions

In the previous section, we found the feasible allocation that maximizes the utility of the future generations. However, to achieve this allocation, in each period the central planner would have to take away c_2^* from each young person and give this amount to each old person. Such a redistribution requires that the central planner have the ability to reallocate endowments costlessly between the generations. Furthermore, in order to determine c_1^* and c_2^*, this central planner also must know the exact utility function of the subjects.

These are strong assumptions about the power and wisdom of central planners. This leads us to ask if there is some way we can achieve this optimal allocation in a more decentralized manner, one in which the economy reaches the optimal allocation through mutually beneficial trades conducted by the individuals themselves. In other words, can we let a market do the work of the central planner?

Before we answer this question, we need to define some terms that are used throughout the book. First, we discuss the notion of a competitive equilibrium. A

competitive equilibrium has the following properties:

1. Each individual makes mutually beneficial trades with other individuals. Through these trades, the individual attempts to attain the highest level of utility that he can afford.
2. Individuals act as if their actions have no effect on prices (rates of exchange). There is no collusion between individuals to fix total quantities or prices.
3. Supply equals demand in all markets. In other words, markets clear.

Equilibrium Without Money

Let us consider the nature of the competitive equilibrium when there is no money in our economy of overlapping generations. Recall that agents are endowed with some of the consumption good when young. Their endowment is zero when old. Their utility can be increased if they give up some of their endowment when they are young in exchange for some of the goods when they are old. Without the presence of an all-powerful central planner, we must ask ourselves if there are trades between individuals in the economy that could achieve this result.

No such trades are possible. Refer to Figure 1.1, which outlines the pattern of endowments. A young person at period t has two types of people with whom to trade potentially in period t – other young people of the same generation or old people of the previous generation. However, trade with fellow young people would be of no benefit to the young person under consideration. They, like him, have none of the consumption good when they are old. Trade with the old would also be fruitless; the old want the good the young have, but they do not have what the young want (because they will not be alive in the next period). The source of the consumption good at time $t+1$ is from the people who are born in that period. However, in period t, these people have not yet come into the world and so do not want what young people have to trade. This lack of possible trades is the manner in which the basic overlapping generations model captures the "absence of double coincidence of wants" [a term introduced by the nineteenth century economist W. S. Jevons (1875) to explain the need for money]. Each generation wants what the next generation has but does not have what the next generation wants.[2]

The resulting equilibrium is **autarkic** – individuals have no economic interaction with others. Unable to make mutually beneficial trades, each individual consumes his entire endowment when young and nothing when old. In this autarkic equilibrium, utility is low. Both the future generations and the initial old are worse off than they would be with almost any other feasible consumption bundle. A member of the future generations would gladly give up some of his endowment when young in order to consume something when old. A member of the initial old would also like to consume something when old.

[2] People cannot trade directly in this model because they are separated in time. The same absence of trade would result if they were separated in space, as in the models of Robert Townsend (1980).

Equilibrium with Money

To open up a trading opportunity that might permit an exit from this grim autarkic equilibrium, we now introduce fiat money into our simple economy. **Fiat money** is a nearly costlessly produced commodity that cannot itself be used in consumption or production and is not a promise to anything that can be used in consumption or production.

For the purposes of our model, we assume the government can produce fiat money costlessly but that it cannot be produced or counterfeited by anyone else. Fiat money can be costlessly stored (held) from one period to the next and it is costless to exchange. Pieces of paper distinctively marked by the government generally serve as fiat money.

Because individuals derive no direct utility from holding or consuming money, fiat money is valuable only if it enables individuals to trade for something they want to consume.

A **monetary equilibrium** is a competitive equilibrium in which there is a valued supply of fiat money. By valued, we mean that the fiat money can be traded for some of the consumption good. For fiat money to have value, its supply must be limited and it must be impossible (or very costly) to counterfeit. Obviously, if everyone has the ability to print money costlessly, its supply will rapidly approach infinity, driving the value of any one unit to zero.

We begin our analysis of monetary economies with an economy with a fixed stock of M perfectly divisible units of fiat money. We assume that each of the initial old begins with an equal number, M/N, of these units.[3]

The presence of fiat money opens up a trading possibility. A young person can sell some of his endowment of goods (to old persons) for fiat money, hold the money until the next period, and then trade the fiat money for goods (with the young of that period).

Finding the Demand for Fiat Money

Of course, this new trading possibility exists only if fiat money is valued – in other words, if people are willing to give up some of the consumption good in trade for fiat money and vice versa. Because fiat money is intrinsically useless, its value depends on one's view of its value in the future, when it will be exchanged for the goods that do increase an individual's utility.

If it is believed that fiat money will not be valued in the next period, then fiat money will have no value in this period. No one would be willing to give up some of the consumption good in exchange for it. That would be tantamount to trading something for nothing.

[3] Because the government is the creator of fiat money, we are implicitly assuming that the government has made a gift of the initial money stock to the initial old.

Extending this logic, we can predict that fiat money will have no value today if it is known with complete certainty that fiat money will be valueless at any future date T. To see this, first ask what the value of fiat money will be at time $T-1$; in other words, ask how many goods you would be willing to give for money at $T-1$ if it is known that it will be worthless at time T. The answer, of course, is that you would not be willing to give up any goods at time $T-1$ for money. In other words, fiat money would have no value at time $T-1$. Then what must its value be at time $T-2$? By similar reasoning, we see that it will also be valueless at time $T-2$. Working backward in this manner, we can see that fiat money will have no value today if it will be valueless at some point in the future.

Now let us consider a more interesting equilibrium where money has a positive value in all future periods. We define v_t as the value of 1 unit of fiat money (let us call the unit a dollar) in terms of goods; that is, it is the number of goods that one must give up to obtain one dollar. It is the inverse of the dollar price of the consumption good, which we write as p_t. For example, if a banana costs 20 cents, $p_t = 1/5$ dollars and the value of a dollar, v_t, is five bananas. Note also that because our economy has only one good, the price of that good p_t can be viewed as the price level in this economy.

An Individual's Budget

Let us now examine how individuals will decide how much money to acquire (assuming that fiat money will have a positive value in the future). To answer, we must first establish the constraints on the choices of the individual – why he cannot simply enjoy infinite consumption both when young and when old. As it was for the entire society, the constraints on an individual are that he cannot give up more goods than he has. We will refer to the limitations on an individual's consumption as his **budget constraints**.

In the first period life, an individual has an endowment of y goods. The individual can do two things with these goods – consume them and/or sell them for money. Notice that no one in the future generations is born with fiat money. To acquire fiat money, an individual must trade. If the number of dollars acquired by an individual (by giving up some of the consumption good) at time t is denoted by m_t, then the total number of goods sold for money is $v_t m_t$. We can therefore write the budget constraint facing the individual in the first period of life as

$$c_{1,t} + v_t m_t \leq y. \tag{1.7}$$

The left-hand side of Equation 1.7 is the individual's total uses of goods (consumption and acquisition of money). The right-hand side of Equation 1.7 represents the total sources of goods (the individual's endowment).

In the second period of life, the individual receives no endowment. Hence, when old, an individual can acquire goods for consumption only by spending the money acquired in the previous period. In the second period of life (period $t+1$), this money will purchase $v_{t+1}m_t$ units of the consumption good. The only use for these goods is second-period consumption. This means that the constraint facing the individual in the second period of life is

$$c_{2,t+1} \leq v_{t+1}m_t. \tag{1.8}$$

In a monetary equilibrium where, by definition, $v_t > 0$ for all t, we can rewrite this constraint as $m_t \geq (c_{2,t+1})/(v_{t+1})$ and substitute it into the first-period constraint (Equation 1.7) to obtain

$$c_{1,t} + \frac{v_t c_{2,t+1}}{v_{t+1}} \leq y, \tag{1.9}$$

or

$$c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y. \tag{1.10}$$

Equation 1.10 expresses the various combinations of first- and second-period consumption that an individual can afford over a lifetime. In other words, it is the individual's **lifetime budget constraint**.

We can graph this budget constraint as shown in Figure 1.7. We can easily verify that the intercepts of the budget line are as illustrated. The budget line represents

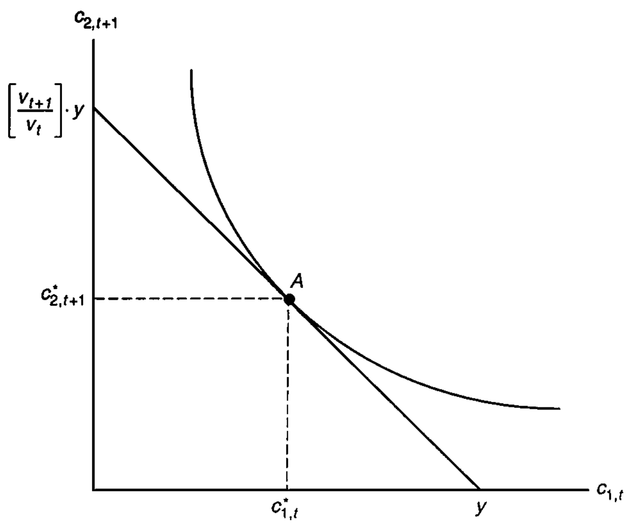

Figure 1.7. The choice of consumption with fiat money. At point A individuals maximize utility given their lifetime budget set in the monetary equilibrium. Point A is found by locating a point of tangency between an indifference curve and the individual's lifetime budget set line. The rate of return on fiat money determines the slope of the budget set line.