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Edited by
N. Ray
and
G. Walker

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Manchester 1990

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Contents of Volume 1

Preface

The international Symposium on algebraic topology which was held in Manchester in July 1990 was originally conceived as a tribute to Frank Adams by mathematicians in many countries who admired and had been influenced by his work and leadership. Preparations for the meeting, including invitations to the principal speakers, were already well advanced at the time of his tragic death in a car accident on 7 January 1989, at the age of 58 and still at the height of his powers.

Those members of the Symposium, and readers of these volumes, who had the good fortune to know Frank as a colleague, teacher and friend will need no introduction here to the qualities of his intellect and personality. Others are referred to Ioan James's article, published as *Biographical Memoirs of Fellows of the Royal Society*, Vol. 36, 1990, pages 3–16, and to the Memorial Address and the Reminiscences written by Peter May and published in *The Mathematical Intelligencer*, Vol. 12, no. 1, 1990, pages 40–44 and 45–48.

We, the editors of these proceedings, were both research students of Frank's during his years at Manchester, As might be imagined, this was a remarkable and unforgettable experience. There was inspiration in plenty, and, on occasion, humble pie to be eaten as well. The latter became palatable as we learned to appreciate that the vigour of Frank's responses was never directed at us as individuals, but rather towards the defence of mathematics. In fact we both discovered that when suitably prompted, Frank was astonishly willing to repeatedly explain arguments that we had bungled. He also provided warm and understanding support, friendship and guidance far beyond his role as research supervisor.

This was an exciting period for Manchester, where Frank's influence was admirably complemented by Michael Barratt, and for algebraic topology in general. When he came to Manchester in 1962, Frank had just developed the K-theory operations he used to solve the problem of vector fields on spheres. In the years that followed he developed his series of papers on $J(X)$, and regularly lectured on subject matter which eventually became his Chicago Lecture Notes volume "Stable homotopy and generalised homology".

Our opening article, by Peter May, describes these and other achievements in more detail, and forms in a sense an introduction to the whole of the book. Although some attempt has been made to group papers according to the themes which May identifies, we cannot pretend that anything very systematic has been attained, or is even desirable. Most of the contributions here are based on talks given at the Symposium, as the reader will see by consulting lists on pages xi–xii. Aside from this, we feel it sufficient to remark that all the articles have been refereed, and that every attempt has been made to attain a mathematical standard worthy of association with the name of J. F. Adams – with what success we must leave the reader to judge.

We also hope that the Symposium itself might be seen as a significant tribute to his philosophy and powers. In keeping with his views on the value of mathematics in transcending political and geographical boundaries, we were fortunate to attract a large number of participants from many countries, including Eastern Europe and the Soviet Union.

In conclusion, we would like to thank the many organisations and individuals who made possible both the Symposium and these volumes.

The bulk of the initial funding was provided by the Science and Engineering Research Council, with substantial additions being made by the London Mathematical Society and the University of Manchester Research Support Fund. Support for important peripherals was given by the NatWest Bank, Trinity College Cambridge, and the University of Manchester Mathematics Department and Vice-Chancellor's Office. We would especially like to thank John Easterling and Mark Shackleton in this context.

During the Symposium our sanity would not have survived intact without the able assistance of all our Manchester students and colleagues in algebraic topology, and most significantly, the fabulous organisational and front-desk skills of the Symposium Secretary, Jackie Minshull. And the high point of the Symposium, an ascent of Tryfan (Frank's favourite Welsh Peak), would have been far less enjoyable without the presence of Manchester guide Bill Heaton.

Mrs Grace Adams and her family were most helpful in providing photographs and other information, and were very supportive of the Symposium despite their bereavement.

The production of these volumes was first conceived by the Cambridge University Press Mathematical Editor David Tranah, and their birth pangs were considerably eased by his laid-back skills. Our referees rose to the task of supplying authoritative reports within what was often a tight deadline. We should also thank those authors who offered a manuscript which we have not had space to publish.

Finally, we both owe a great debt to our respective families, for sustaining us throughout the organisation of the Symposium, and for continuing to support us as its ripples spread downwards through the following months. Therefore, to Sheila Kelbrick and our daughter Suzanne, and to Wendy Walker, thank you.

These volumes are dedicated to Frank's memory.

Nigel Ray Grant Walker

University of Manchester
September 1991

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Programme of one-hour invited lectures

A. K. Bousfield	On K_*-local stable homotopy theory.
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F. W. Clarke	Cooperations in elliptic homology.
M. C. Crabb	The Adams conjecture and the J map.
E. S. Devinatz	Duality in stable homotopy theory.
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P. Goerss	Projective and injective Hopf algebras over the Dyer-Lashof algebra.
M. J. Hopkins	p-adic interpolation of stable homotopy groups.
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J. D. S. Jones	Morse theory and classifying spaces.
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M. E. Mahowald	On the tertiary suspension.
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D. C. Ravenel	The telescope conjecture.
C. A. Robinson	Ring spectra and the new cohomology of commutative rings.
Y. Rudjak	Orientability of bundles and fibrations and related topics.
V. Vershinin	The Adams spectral sequence as a method of studying cobordism theories.
C. W. Wilkerson	Lie groups and classifying spaces.

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J. R. Hunton	Detruncating Morava K-theory.
S. Hutt	A homotopy theoretic approach to surgery on Poincaré spaces.
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M. C. Tangora	The theorems of Poisson, Euler and Bernoulli on the Adams spectral sequence.
C. B. Thomas	Characteristic classes of modular representations.
R. M. W. Wood	The boundedness conjecture for the action of the Steenrod algebra on polynomials.

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D. Arlettaz: The Hurewicz homomorphism in dimension 2.

M. Beattie: Proper suspension and stable proper homotopy groups.

T. Bisson: Covering spaces as geometric models of cohomology operations.

D. Blanc Operations on resolutions and the reverse Adams spectral sequence.

J. M. Boardman: Group cohomology and gene splitting.

P. Booth: Cancellation and non-cancellation amongst products of spherical fibrations.

C. Casacuberta and M. Pfenniger: On orthogonal pairs in categories and localization.

S. Edwards: Complex manifolds with c_1 non-generating.

V. Franjou: A short proof of the $\mathcal{U}$-injectivity of H^*RP^∞.

V. G. Gorbunov: Symplectic bordism of projective spaces and its application.

T. Hunter: On Steenrod algebra module maps between summands in $H^*((Z/2)^s; F_2)$.

K. Ishiguro Classifying spaces of compact simple Lie groups and p-tori.

N. Iwase: Generalized Whitehead spaces with few cells.

M. Kameko: Generators of $H^*(RP^\infty \times RP^\infty \times RP^\infty)$.

S. Kochman: Lambda algebras for generalized Adams spectral sequences.

I. Leary and N. Yagita: p-group counterexamples to Atiyah's conjecture on filtration of $R_C(G)$.

A. T. Lundell Concise tables of homotopy of classical Lie groups and homogeneous spaces.

G. Moreno: Lower bounds for the Hurewicz map and the Hirzebruch Riemann-Roch formula.

R. Nadiradze Adams spectral sequence and elliptic cohomology.

N. Oda: Localisation of the homotopy set of the axes of pairings.

A. A. Ranicki: Algebraic L-theory assembly.

N. Ray: Tutte algebras of graphs and formal groups.

J. Rutter: The group of homotopy self-equivalence classes of non-simply connected spaces: a method for calculation.

C. R. Stover On the structure of $[\Sigma\Omega\Sigma X, Y]$, described independently of choice of splitting $\Sigma\Omega\Sigma X \longrightarrow \bigvee_{n=1}^{\infty} \Sigma X^{(n)}$.

P. Symonds: A splitting principle for group representations.

Z. Wojtkowiak: On 'admissible' maps and their applications.

K. Xu: Representing self maps.

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The Work of J. F. Adams[1]

I first met Frank here in Manchester in 1964, when this building was being planned. I remember from the first feeling that he was a far more impressive man than the anecdotes of his exploits had led me to expect, and a far nicer one. I also felt humbled by the sheer amount of mathematics that he knew and perhaps more so by the amount that he somehow assumed I knew. I feel a little the same way now, faced with this audience and this topic. Still, I don't want to spend much time in reminiscence.[2] I want rather to give a quick guided tour through Frank's work, largely letting it speak for itself.

I should say that Frank's collected works are to be published in the near future by the Cambridge University Press. Like this talk, the collected works are organized by subject matter rather than by strict chronology. However, I will begin not quite at the beginning of his work with a sequence of four papers submitted between 1955 and 1958. All dates cited are dates of submission, not necessarily of appearance.

A. The cobar construction, the Adams spectral sequence, higher order cohomology operations, and the Hopf invariant one problem

1. *On the chain algebra of a loop space* (1955, with Peter Hilton) [**5**][3]
2. *On the cobar construction* (1956) [**6**]

Let K be a CW-complex with trivial 1-skeleton. In the first paper, a DGA-algebra $A(K)$ is constructed whose homology is the Pontryagin algebra $H_*(\Omega K)$; as an algebra, $A(K)$ is free on generators in bijective correspondence with the cells of K (other than the vertex). As Kathryn Hess explained in her talk a few hours ago, this Adams-Hilton model is small enough to be of concrete value for computations and is still being used and studied today. In the second paper, a larger, but functorial, DGA is given whose homology is $H_*(\Omega K)$, namely the cobar construction $F(C_*(K))$. This construction was discussed in John McCleary's talk on Hochschild homology. Nowadays, an obvious and trivial next step after the introduction of

[1]Reconstruction and expansion of the talk given at the conference, most of which was not written out beforehand

[2]A more personal tribute has been published in The Mathematical Intelligencer, Vol. 12, No. 1, 1990, 40–48.

[3]Details of publication of Adams' works discussed here can be found in the complete bibliography which follows this paper.

the cobar construction would be to filter it and so arrive at what is called the Eilenberg-Moore spectral sequence for the computation of $H_*(\Omega K)$. In fact, Moore and Adams were already in contact before this paper was written, and it was cited by Eilenberg and Moore as an important precursor to their work.

3. *On the structure and applications of the Steenrod algebra* (1957) [9]

Adams viewed this paper as a step towards the solution of the Hopf invariant one problem. The main theorem states that if $\pi_{2n-1}(S^n)$ and $\pi_{4n-1}(S^{2n})$ both contain elements of Hopf invariant one, then $n \leq 4$. It is now chiefly celebrated for the introduction of the Adams spectral sequence converging from $\mathrm{Ext}_A^{s,t}(H^*(X), Z_p)$ to ${}_p\pi_*^s(X)$. Products are defined in the spectral sequence when $X = S^0$, and the sub-Hopf algebras A_r are used to compute products of the elements h_i inductively, where h_i corresponds to Sq^{2^i}. The basic argument runs as follows. Let $n = 2^m$, $m \geq 3$. Assuming that h_m is a permanent cycle, $h_0(h_m)^2$ would survive to E_∞ if $d_2 h_{m+1} = 0$. This would contradict the fact that, in $\pi_*^s(S^0)$, $2x^2 = 0$ if $\deg(x)$ is odd. This seems straightforward enough today, but it was revolutionary at the time. The idea of reducing such a fundamental topological problem as Hopf invariant one to the non-triviality of a particular differential in a spectral sequence was quite new and unexpected.

Adams was curiously modest about the Adams spectral sequence. He always referred to it as a formalization of the Cartan-Serre method of killing homotopy groups. I think we all see it as something very much more than that. Its introduction was a watershed, and it substantially raised the level of algebraic sophistication of our subject.

4. *On the non-existence of elements of Hopf invariant one* (1958) [**14**]

If $\pi_{2n-1}(S^n)$ contains an element of Hopf invariant one, then $n = 1, 2, 4$, or 8. The proof is based on showing that Sq^{2^m} decomposes in terms of secondary cohomology operations if $m > 3$. The paper contains definitive homological algebra for the study of Ext_A, including minimal resolutions and the cobar construction with its $\smile$ and $\smile_1$ products. It uses Milnor's description of A^* to redo the calculations in the previous paper. It gives a detailed study of stable secondary cohomology operations via universal examples, which are generalized two-stage Postnikov systems. The results include axioms for the operations, existence and uniqueness theorems, the relationship between the operations and Tor_A^2, and a Cartan formula. Particular operations are studied via homological algebra, and a key computation in $\mathbf{C}P^\infty$ is used to start the induction which shows that the undetermined constants in the decompositions of the Sq^{2^m} are non-zero.

Adams was a problem solver. He introduced exactly the tools he needed to solve the problems he studied, and he had relatively little interest in Bourbaki style analysis of the foundations or in systematic calculations. He had an extraordinary talent for proving important and easily formulated conceptual theorems through a mix of new ideas, new foundational constructions, and adroit calculations. The solution of the Hopf invariant one problem was the first of many such successes.

B. Applications of K-theory

1. *Vector fields on spheres* (1961) [**23**]

Having so spectacularly solved the Hopf invariant one problem, Adams turned next to the vector fields problem. It was natural for him to try cohomology operations here too. A 1960 note [**20**] gave a partial result, and he was still working in cohomology in July, 1961, when he gave a series of lectures in Berkeley. When the solution came, however, it used K-theory and Adams wrote of his cohomological efforts: "The author's work on this topic may be left in decent obscurity, like the bottom nine-tenths of an iceberg." Write $n = (2a+1)2^b$ and $b = c + 4d$ and let $\rho(n) = 2^c + 8d$. Hurwicz-Radon and Eckman had shown that there exist $\rho(n) - 1$ linearly independent vector fields on S^{n-1}. Adams proved that there do not exist $\rho(n)$ such fields. It suffices to show that the truncated projective space $\mathbf{R}P^{m+\rho(m)}/\mathbf{R}P^{m-1}$ is not coreducible (the bottom cell is not a retract up to homotopy) for any m. He introduced what are now called the Adams operation ψ^k into real and complex K-theory, he calculated the K-theory of truncated projective spaces, with their Adams operations, and he showed that there is no splitting of their real K-theory which is compatible with the operations. All of Adams' papers are well written, but the exposition in this classic paper is especially lovely.

For background, James, in part, and Atiyah had shown that the bundle $O(n)/O(n-k) \to S^{n-1}$ admits a cross-section if and only if n is a multiple of the order of the image of the canonical line bundle in $\tilde{J}(\mathbf{R}P^{k-1})$, and analogously in the complex and quaternionic cases. Curiously, it was left to Atiyah and Bott to observe that Adams' calculations actually imply that $\widetilde{KO}(\mathbf{R}P^k) \cong \tilde{J}(\mathbf{R}P^k)$. This group is cyclic of order $2^{\varphi(k)}$, where $\varphi(k)$ is the number of j such that $0 < j \leq k$ and $j \equiv 0, 1, 2$, or $4 \bmod 8$. Mark Mahowald discussed the significance of this calculation in his talk.

2. *On complex Stiefel manifolds* (1964, with Grant Walker) [**29**]

It is shown that $U(n)/U(n-k) \to S^{2n-1}$ admits a cross-section if and only if M_k divides n; here $\nu_p(M_k) = \sup\{r+\nu_p(r)\,|\,1 \le r \le (k-1)/(p-1)\}$ if $p \le k$ and $\nu_p(M_k) = 0$ if $p > k$. Atiyah and Todd had shown that the condition is necessary, and they had conjectured that it is sufficient. As already noted, Atiyah had reduced the problem to a calculation in $\tilde{J}(CP^{k-1})$, and this paper analyzes $\tilde{J}(CP^n)$ by the methods of $J(X)$-I,II. It gives a worked example of the general study in those papers.

3. *On the groups* $J(X)$*-I* (1963), *II* (1963), *III* (1963), *IV* (1965) ([**25**], [**28**], [**31**], [**35**])

The program in this fundamentally important cycle of papers is to give effective means for computing the group $J(X) = \tilde{J}(X) \oplus Z$ of fiber homotopy equivalence classes of stable vector bundles over a finite CW-complex X. The basic idea is to give computable upper and lower bounds $J''(X)$ and $J'(X)$ for $J(X)$ and to show that the two bounds coincide. Thus $J(X)$ would be captured in the diagram of epimorphisms

$$\begin{tikzcd} & J''(X) \arrow[d] \\ KO(X) \arrow[ur] \arrow[r] \arrow[dr] & J(X) \arrow[d] \\ & J'(X). \end{tikzcd}$$

That $J''(X)$ really is an upper bound depends on the celebrated Adams conjecture: "If k is an integer, X is a finite CW-complex and $y \in KO(X)$, then there exists a non-negative integer $e = e(k, y)$ such that $k^e(\psi^k - 1)y$ maps to zero in $J(X)$."

As Michael Crabb explained in his talk, it is now possible to give a fairly elementary proof of the Adams conjecture. It is fortunate that such an argument was not discovered early on. The proofs of the Adams conjecture by Sullivan and Quillen led to a veritable cornucopia of new mathematics, including localizations and completions of spaces and the higher algebraic K-groups of rings.

J(X)-I. The Adams conjecture is proven if y is a linear combination of $O(1)$ and $O(2)$ bundles or if $X = S^{2n}$ and y is a complex bundle. The proof is based on the Dold theorem mod k: if there is a fiberwise map $E_\xi \to E_\eta$ of degree $\pm k$ on each fiber, then $k^e\xi$ and $k^e\eta$ are fiber homotopy equivalent for some $e > 0$.

J(X)-II. The group $J''(X)$ is specified as $KO(X)/W(X)$, where $W(X)$ is the subgroup generated by all elements $k^{e(k)}(\psi^k - 1)y$ for a suitable function e (independent of y). The cannibalistic classes ρ^k are defined by the formula $\rho^k(\xi) = \varphi^{-1}\psi^k\varphi(1)$ on Spin(8n)-bundles ξ, and it is shown that they can be defined more generally after localization. If ξ and η are fiber homotopy equivalent, then $\rho^k(\xi) = \rho^k(\eta)[\psi^k(1+y)/(1+y)]$ for some $y \in \widetilde{K}(X)$ (independent of k). $J'(X)$ is specified as $KO(X)/V(X)$, where $V(X)$ is the subgroup of those x such that $\rho^k(x) = \psi^k(1+y)/(1+y)$ in $KO(X)\otimes Z[1/k]$ for all $k \neq 0$ and some $y \in \widetilde{K}O(X)$. Explicit computations give the groups $KO(\mathbf{R}P^n) = J''(\mathbf{R}P^n) = J'(\mathbf{R}P^n)$ and $J''(S^n) = J'(S^n)$. The latter calculations imply that $J(\pi_{8n+i}(SO)) = Z_2$ if $i = 0$ or 1 and that $J(\pi_{4n-1}(SO))$ is cyclic of order $m(2n)$, where $m(2n)$ is the denominator of $B_n/4n$, although Adams was left with an ambiguity when n is even because he only had the complex and not the real Adams conjecture for bundles over spheres. Of course, these basic calculations are essential to the understanding of the stable homotopy groups of spheres.

J(X)-III. The main theorem of the series is proven: $J'(X) = J''(X)$. This is based on the fundamental commutative diagram

$$\begin{array}{ccc}
\sum_k \widetilde{K}SO(X) & \xrightarrow{\sum k^{e(k)}(\psi^k-1)} & \widetilde{K}SO(X) \\
\big\downarrow{\scriptstyle \sum \vartheta^k} & & \big\downarrow{\scriptstyle \prod \rho^\ell} \\
1+\widetilde{K}SO(X) & \xrightarrow{\prod \psi^\ell/1} & \prod_\ell 1+\widetilde{K}SO(X)\otimes Z[1/\ell]
\end{array}$$

The diagram is obtained by summing individual diagrams for pairs (k,ℓ), and the ϑ^k are constructed in the course of the character theoretic proof. The main theorem follows from the fact that this diagram is a weak pullback. The paper also explains and exploits the modular periodicity of the Adams operations.

The paper has a tantalizing last section. It asks for a theory Sph(X) of stable spherical fibrations in which $J(X)$ is a direct summand mapped to by $KO(X)$; Sph(X) should be represented by $BF \times Z$, where F is the monoid of homotopy equivalences of spheres. It also asks for a theory Sph($X; kO$) of kO-oriented stable spherical fibrations and gives a number of probable consequences. With characteristic honesty, Adams wrote of this discussion "I will not call the results "theorems", since the underlying assumptions have not been stated precisely enough." This section makes vividly clear just how prescient this whole series of papers was. Many relevant and now standard tools were unavailable to Adams, but he foresaw much that would later be formulated and proven with them.

For example, an alternative version of the diagram above can be constructed conceptually by exploiting localized classifying spaces rather than representation theory. At $p = 2$, the relevant diagram is:

$$\begin{tikzcd}
BO \arrow[d, "\gamma^3"] \arrow[dr, "\psi^3 - 1"] & & \\
SF/\mathrm{Spin} \arrow[r] \arrow[d, "\nu"] & B\,\mathrm{Spin} \arrow[r, "J"'] \arrow[d, "\mu"] & BSF \arrow[d, equal] \\
BO_{\otimes} \arrow[r] \arrow[dr, "\psi^3/1"'] & B(SF; kO) \arrow[r] \arrow[d, "c(\psi^3)"] & BSF \\
 & B\,\mathrm{Spin}_{\otimes} &
\end{tikzcd}$$

Here $B(SF; kO)$ classifies $\mathrm{Sph}(F; kO)$, $c(\psi^3)$ is the universal cannibalistic class determined by ψ^3, and μ is given by the Atiyah-Bott-Shapiro orientation. The rows are fibration sequences, so μ determines ν and the Adams conjecture determines γ^3. The composite $c(\psi^3)\circ\mu$ is ρ^3, the composite $\nu\circ\gamma^3$ can be taken as ϑ^3, and these two maps are 2-local equivalences.[4] I discussed this approach with Frank, who had envisioned something of the sort. He very much liked it, but he rightly emphasized that you can't proceed this way before you have the Adams conjecture.

J(X)-IV. The results of I–III are applied to computations in the stable homotopy groups of spheres. The starting point is an abstract analysis of the "d and e invariants" of a half exact functor k from the homotopy category to an abelian category $\mathcal{A}$. For $f : X \to Y$, $d(f) = f^* \in \mathrm{Hom}(k(Y), k(X))$. If $d(f) = 0$ and $d(\Sigma f) = 0$, then $e(f)$ is the class of

$$0 \to k(\Sigma X) \to k(Cf) \to k(Y) \to 0$$

in $\mathrm{Ext}^1(k(Y), k(\Sigma X))$. In the applications, $\mathcal{A}$ consists of finitely generated Abelian groups with Adams operations, k is taken to be $\widetilde{K}$ or $\widetilde{KO}$, and X and Y are taken to be spheres or Moore spaces, for which the Hom and Ext target groups are readily computed.

[4]For details, see Chapter V of [J. P. May (with contributions by Nigel Ray, Frank Quinn, and J. Tornehave) E_∞ ring spaces and E_∞ ring spectra. Springer Lecture Notes in Mathematics Vol 577, 1977].