NONCIRCULAR GEARS

Design and Generation

Faydor L. Litvin • Alfonso Fuentes-Aznar
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Noncircular Gears: Design and Generation represents the extension of the modern theory of gearing applied to the design and manufacture of the main types of noncircular gears: conventional and modified elliptical gears, eccentric gears, oval gears, gears with lobes, and twisted gears. This book is enhanced by updated theoretical descriptions of the methods of generation of noncircular gears by enveloping methods similar to those applied to the generation of circular gears. *Noncircular Gears: Design and Generation* also offers new developments intended to extend the application of noncircular gears for output speed variation and generation of functions. Numerous numerical examples show the application of the developed theory. This book aims to extend the application of noncircular gear drives in mechanisms and industry.

Faydor L. Litvin has been a Professor at the University of Illinois at Chicago for the past 30 years, after 30 years as a Professor and Department Head of Leningrad Polytechnic University and Leningrad Institute of Precise Mechanics and Optics. Dr. Litvin is the author of more than 300 publications (including 10 monographs) as well as the inventor and co-inventor of 25 inventions. Among his many honors, Dr. Litvin was made Doctor Honoris Causa of Miskolc University, Hungary, in 1999. He was named Inventor of the Year 2001 by the University of Illinois at Chicago and has been awarded 12 NASA Tech Brief awards; the 2001 Thomas Bernard Hall Prize (Institution of Mechanical Engineers, UK); and the 2004 Thomas A. Edison Award (ASME). He was elected a Fellow ASME and is an American Gear Manufacturers Association (AGMA) member. He has supervised 84 Ph.D. students. In addition to his deep interest in teaching, Dr. Litvin has conducted seminal research on the theory of mechanisms and the theory and design of gears.

Alfonso Fuentes-Aznar is a Professor of Mechanical Engineering at the Polytechnic University of Cartagena (UPCT). He has more than 15 years of teaching experience in machine element design and is the author of more than 65 publications. He was a Visiting Scholar and Research Scientist at the Gear Research Center of the University of Illinois at Chicago from 1999 to 2001. Among his many honors, he was awarded the 2001 Thomas Bernard Hall Prize (Institution of Mechanical Engineers, UK) and the NASA Tech Brief Award No. 17596-1 for the development of a new technology – "New Geometry of Face Worms Gear Drives with Conical and Cylindrical Worms." He is a member of the editorial boards of *Mechanism and Machine Theory*, the *Open Mechanical Engineering Journal*, and *Recent Patents on Mechanical Engineering*. Dr. Fuentes-Aznar is a member of AGMA and ASME. This is his second book.

Ignacio Gonzalez-Perez earned his Ph.D. from the Polytechnic University of Cartagena. He was a Visiting Scholar at the Gear Research Center of the University of Illinois at Chicago, from 2001 to 2003 and a Visiting Scientist at the Gear Research and Development Department of Yamaha Motor Co., Ltd., in Japan, in 2005. Dr. Gonzalez-Perez is currently an Associate Professor in the Department of Mechanical Engineering of the Polytechnic University of Cartagena. He is a member of AGMA and ASME. He received an AGMA student paper award in 2002 and a UPCT research award for his thesis in 2006.

Kenichi Hayasaka graduated from Tohoku University, Japan, in 1977 and earned his master's degree in 1983 from the University of Illinois at Chicago. He is the author of 16 research publications and inventor and co-inventor of 23 inventions. Mr. Hayasaka has been a researcher and engineer at Yamaha Motor Co., Ltd., since 1977. He is involved in the research and design of gears and transmissions applied in motorcycles and marine propulsion systems, and in the development of enhanced computerized design systems for optimizing low noise and durability of spiral bevel gears, helical gears, and spur gears.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521761703

First published in print format 2009

ISBN-13 978-0-511-60197-2 eBook (Adobe Reader)

ISBN-13 978-0-521-76170-3 Hardback

To my followers:

When I'll be very far,
And look at you from a twinkling star,
I'll whisper (will you hear?)
I love you, my very dear.

FAYDOR L. LITVIN

Contents

Foreword

Noncircular gears (NCG) have been considered a curiosity and a product of niche applications for a long time because of their design and manufacture complexity. However, thanks to the availability of powerful computers and sophisticated CNC machine tools, the design and manufacturing of NCG became more feasible and their potential in many fields can be more easily exploited.

NCG are used to improve the function, versatility, and simplicity of many mechanical processes. For instance, they allow speed matching on assembly lines, linear motion with quick return, and stop-and-dwell motion. Very recent publications highly respected scientific journals show that the interest in NCG is still very strong, for both the theoretical and manufacturing challenges involved, especially in today's high-performance automatic machines.

In this context, this book on NCG is very welcome. It covers all the basic issues of NCG and shows how to solve most of the key problems in NCG design and manufacturing. It clearly presents the foundations of this topic, very useful to the beginner, as well as new and advanced concepts of great importance to both scientists involved in research issues and skillful designers facing NCG sizing and manufacturing. In particular, the book represents the most comprehensive and authoritative treatise on NCG available today for scientists and designers. When complemented with a recently published book, *Gear Geometry and Applied Theory* (2nd ed.) by F. L. Litvin and A. Fuentes (Cambridge University Press, 2004), which provides the fundamentals of all types of gears, the whole field of the theory and applications of gears is covered.

This book on NCG is a bridge between the past and the future of this advanced topic, representing the extension of the existing theory of gearing as it has been developed by Prof. Litvin and his co-authors. In particular, the main contributions of the book can be summarized as follows:

(a) A new algorithm is presented for generation of a given function by any number of gear pairs. It allows more favorable shapes of gear centrodes to be obtained. The analytical solution of this topic did not exist previously.

(b) Design of many types of noncircular gear drives comprising conventional and modified elliptical gears, eccentric circular gears, oval gears, gears with lobes, or internal noncircular gears is covered and their methods of generation by rack cutter, hob, or shaper presented.
(c) New approaches for the analytical design of centrodes are presented, such as modified elliptical centrodes applied for external and internal gear drives. Most of the transmission functions are represented analytically, allowing conditions for avoiding undercutting and interference to be obtained.
(d) Finally, tandem design as a combination of a linkage and a gear drive with NCG, as represented in the book, extends the possibility of variation of the output velocity.

Of Prof. Litvin's life and scientific activity much has been said. In particular, after the most striking sentence reported by John J. Coy (*Mechanism and Machine Theory*, Vol. 30, n. 3., pp. 491–492, 1995) in celebration of the eighty-year milestone of Prof. Litvin, which enlightens his extraordinary personality, not too much remains to be added. Indeed, Dr. Coy reported: "At the time when most people are content to put up their feet and rest, Dr. Faydor Litvin has other things he would rather do. . . . He is always quick to thank others for their inspiration and support. He is grateful for the health and vigor that has enabled his continued contribution." Now, at the age of 95, he and his co-authors present this new and needed book.

I am not the first to believe that Prof. Litvin is an exceptional scientist and a talented man. He faced the writing of this book with the enthusiasm of a young researcher preparing his first paper, but with the wisdom and high depth of this respectable age spent studying and advancing the modern theory of gears.

The reader will be happy and soon fascinated by this book, and will certainly owe Prof. Litvin and his co-authors for such a remarkable piece of work.

Prof. Vicenzo Parenti-Castelli
University of Bologna, Italy

Preface

The book is written by a group of authors united by application of the same methodology and experience of cooperation for a long time. The contents of the book cover:

(1) Methods of generation of noncircular gears by enveloping methods that are similar to those applied for generation of circular gears. However, the motions of the generating tools (rack cutter, shaper, or hob) are nonlinear and must be computerized.
(2) Design of noncircular gears to be applied for variation of output speed and generation of functions. Such a design requires determination of mating centrodes that roll over each other.
(3) Detailed procedure of design of elliptical gears with spur and helical teeth, oval gears, lobes, eccentric involute gears, and twisted gears (applied for extension of the interval of function generation).
(4) Tandem design of planar linkages coupled with noncircular gears for a broader variation range of the output speed.

The authors hope that this book will allow extension of application and design of noncircular gear drives in mechanisms and industry.

Faydor L. Litvin
Alfonso Fuentes-Aznar
Ignacio Gonzalez-Perez
Kenichi Hayasaka

Acknowledgments

The authors express their deep gratitude to institutions, companies, and colleagues that have supported their research. Special thanks to Mr. Kenji Yukishima and Mr. Hiroyuki Nagamoto for their valuable contributions to the preparation of this book.

1 Introduction to Theory of Gearing, Design, and Generation of Noncircular Gears

1.1 Historical Comments

Designers have tried for many years to apply noncircular gears in automatic machines and instruments. The obstacle was the lack of effective methods of generation of noncircular gears similar to those applied for the generation of circular gears. However, researchers have continued the investigation of application of noncircular gears – see the earlier works by Burmester (Burmester, 1888), Golber (Golber, 1939), Temperley (Temperley, 1948), or Boyd (Boyd, 1940) – and manufacturers have intensified their efforts for improvement of the generation of noncircular gears (Fellows, 1924; Bopp & Reuther G.m.b.H., 1938).

Due to the lack of exact methods of generation of noncircular gears, the efforts were first directed to the development of methods based on the meshing of generating tools with master gears. Figure 1.1.1 shows the Fellows' approach where the noncircular master gear 1 is in mesh with a master rack (Fellows, 1924). The rack cutter and gear being generated are denoted by 3 and 4, respectively.

Bopp and Reuther's approach (Fig. 1.1.2) is based on the simulation of meshing of a noncircular master worm gear c with a worm f that is identical to the hob d; a is the spur noncircular gear being generated; the cam b and the follower e form the cam mechanism designated for simulation of the required variable distance between c and f (Bopp & Reuther G.m.b.H., 1938). Weight g maintains the continuous contact between the cam and the follower. However, both approaches were difficult to apply due to the necessity of manufacturing noncircular master gears, which was expensive and time-consuming.

The breakthrough of generation of noncircular gears happened in years 1949–1951, wherein enveloping methods of generation of noncircular gears were developed based on generation by a rack cutter, hob, or a shaper (Litvin *et al.*, 1949 to 1951). Such methods were based on obtaining the gear tooth surface as the envelope to the family of tool surfaces and are based on the following ideas:

(a) The noncircular gears are generated by the same tools (rack cutters, hobs, and shapers) that are used for the manufacture of circular gears.

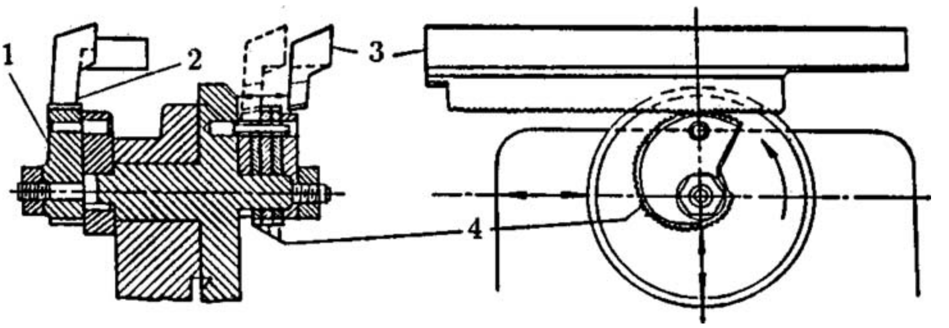

Figure 1.1.1. Generation of a noncircular gear by application of (i) a master rack 2; (ii) a master noncircular gear 1; (iii) a cutting rack cutter 3. The noncircular gear being generated is 4.

(b) Conjugated tooth shapes for noncircular gears are provided due to the imaginary rolling of the tool centrode over the given gear centrode.
(c) The imaginary rolling of the tool centrode over the centrode of the gear being generated is accomplished by proper relations between the motions of the tool and the gear in the process of cutting.

We illustrate the developed approaches in Fig. 1.1.3, which shows that mating noncircular centrodes 1 and 2 are in mesh with a conventional rack cutter 3. The centrode of the rack cutter is a straight line $\overline{t-t}$ that is a common tangent to centrodes 1 and 2 and rolls over 1 and 2. Rolling of centrode 3 of the rack cutter is

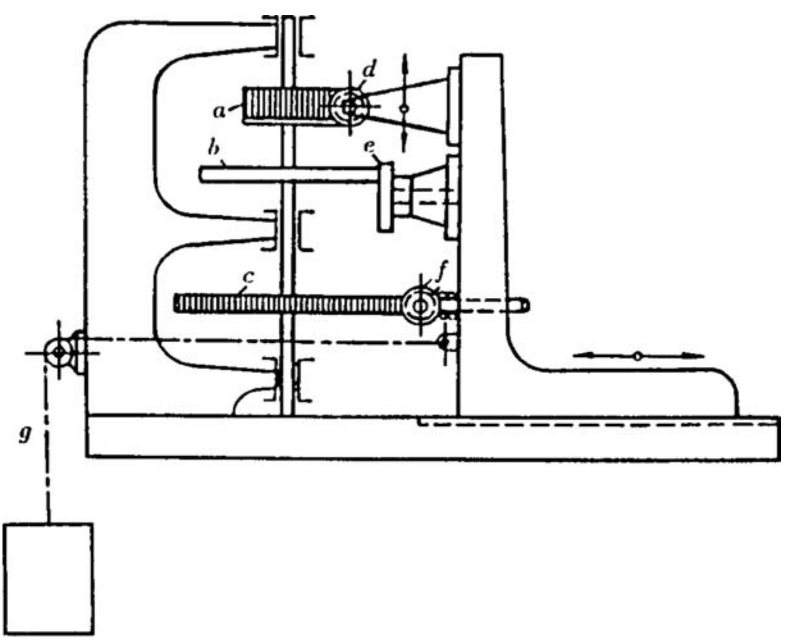

Figure 1.1.2. Generation of a noncircular gear by applying (i) a master worm gear c being in mesh with worm f; (ii) a cam b and follower e; (iii) a is the gear being generated; (iv) d is the hob.

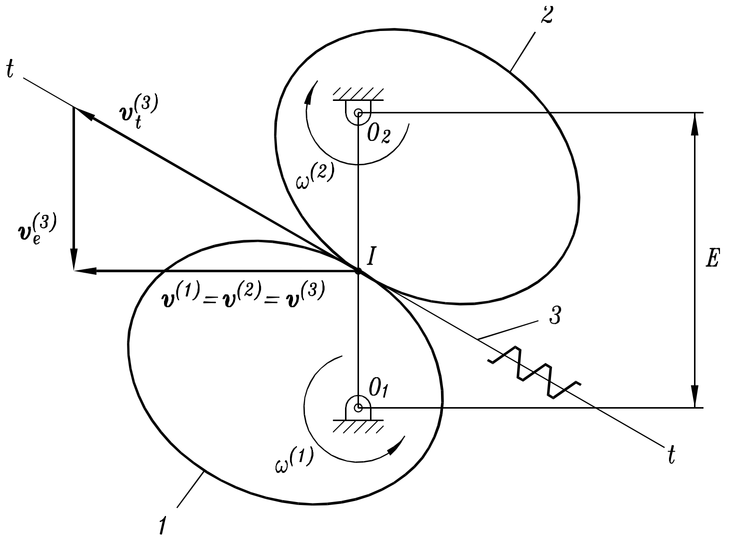

Figure 1.1.3. Illustration of generation of noncircular gears 1 and 2 by rack cutter 3.

achieved wherein the rack cutter translates along tangent $\overline{t-t}$ and is rotated about the instantaneous center of rotation I (Fig. 1.1.3). Tooth surfaces of gear 1, gear 2, and rack cutter 3 are in mesh simultaneously, and gear 1 and 2 are provided with conjugated surfaces.

Drawings of Fig. 1.1.4 show the related motions of a rack cutter being in mesh with one of the noncircular gears of the pair of noncircular gears shown in Fig. 1.1.3. Figure 1.1.4(a) shows that the rack cutter translates along $\overline{t-t}$, which is the common tangent to the rack cutter centrode 2 and centrode 1 of the noncircular gear. Centrode 2 is a straight line. Centrode 1 applied for generation is the same as that applied in the meshing of two mating noncircular gears, as shown in Fig. 1.1.3.

The noncircular gear 1 being in mesh with rack cutter 2 performs two related motions during the process of generation (Fig. 1.1.4(a)): (a) rotation about O_1 with angular velocity $\boldsymbol{\omega}^{(1)}$, and (b) translational motion with linear velocity $\mathbf{v}_{tr}^{(1)}$ in a direction that is perpendicular to $\overline{t-t}$. Rolling of the rack cutter 2 about centrode 1 is provided by observation of the vector equation

$$\mathbf{v}^{(2)} = \mathbf{v}_{rot}^{(1)} + \mathbf{v}_{tr}^{(1)} \tag{1.1.1}$$

We may consider that the translation of the rack cutter is performed with a constant velocity $\mathbf{v}^{(2)}$, and the motions of the noncircular gear are provided by observation of the nonlinear function $\boldsymbol{\omega}^{(1)}(\mathbf{v}^{(2)})$ and $\mathbf{v}_{tr}^{(1)}(\mathbf{v}^{(2)})$.

Three coordinate systems are applied for generation: (i) movable ones, S_2 and S_1, rigidly connected to the rack cutter 2 and noncircular gear 1 (Fig. 1.1.4(b)), and (ii) fixed coordinate system S_f, in which we consider the motions of S_2 and S_1 (Fig. 1.1.4(b)). Symbols $x_f^{(O_2)}$ and $y_f^{(O_1)}$ denote the displacements of the rack cutter and noncircular gear, respectively. Angle ϕ_1 denotes the rotation of the gear.

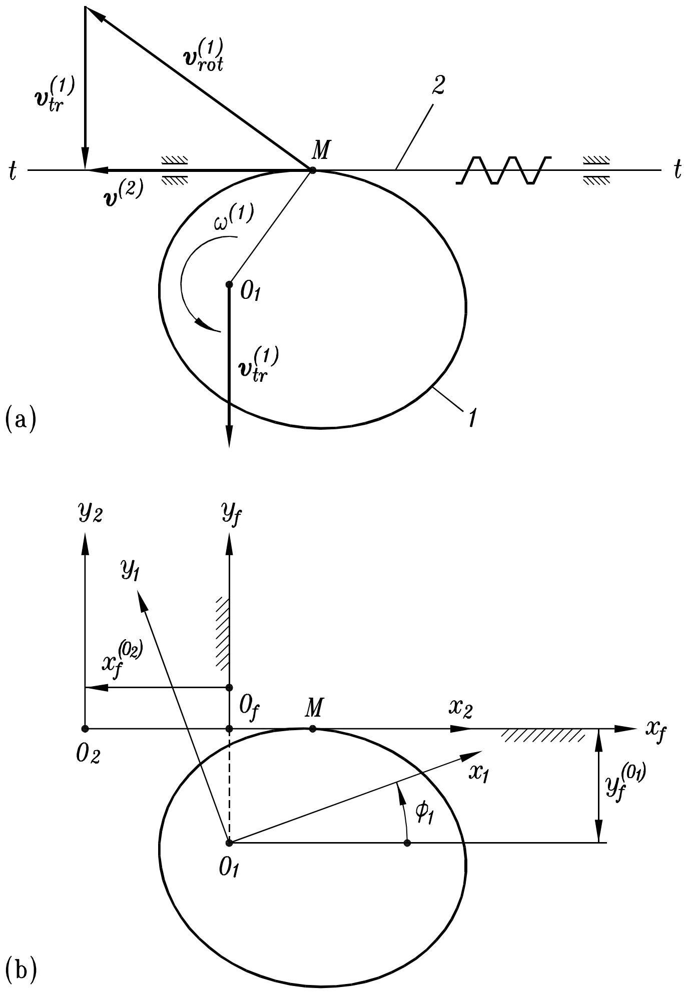

Figure 1.1.4. Toward derivation of related motions of rack cutter 2 and noncircular gear 1.

The derivations of the nonlinear functions $\phi_1(x_f^{(O_2)})$ and $y_f^{(O_1)}(x_f^{(O_2)})$ are presented in Chapter 5. Initially, observation of functions mentioned previously has been accomplished by the remodeling of existing equipment and using cam mechanisms for the generation of the required functions. Figure 1.1.5 shows the first cutting machine for noncircular gears applied in 1951. At present, observation of functions $\phi_1(x_f^{(O_2)})$ and $y_f^{(O_1)}(x_f^{(O_2)})$ is obtained by computer controlled machines (Smith, 1995).

By using enveloping methods of generation of noncircular gears, various new types of noncircular gears have been developed with closed and non-closed centrodes. An example of a pair of noncircular gears with non-closed centrodes applied for generation of a given function $y(x)$ represented in the closed interval $[x_1, x_2]$, if $y(x_1) \neq y(x_2)$ (see Chapter 10), is shown in Fig. 1.1.6.

Figure 1.1.7 shows a 3D model of a helical elliptical gear. It has found good application for the driving of a crank-slider mechanism in heavy-press machines.

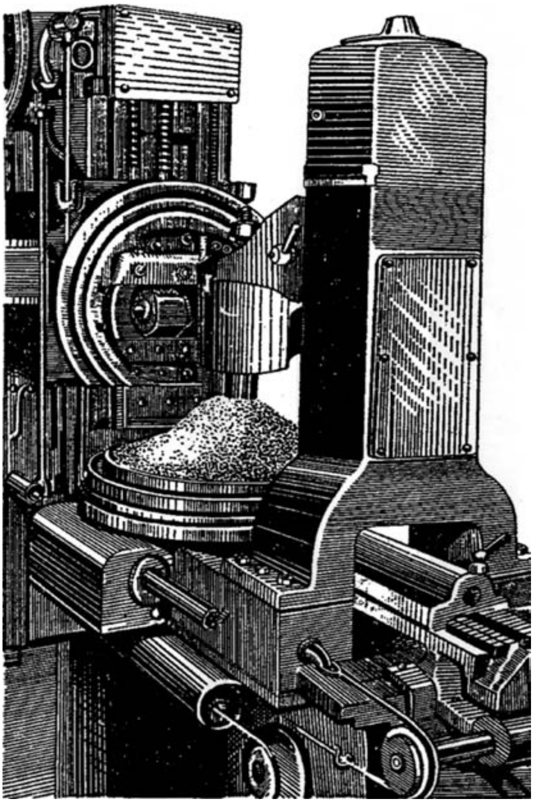

Figure 1.1.5. Remodeled cutting machine for generation of noncircular gears (1951).

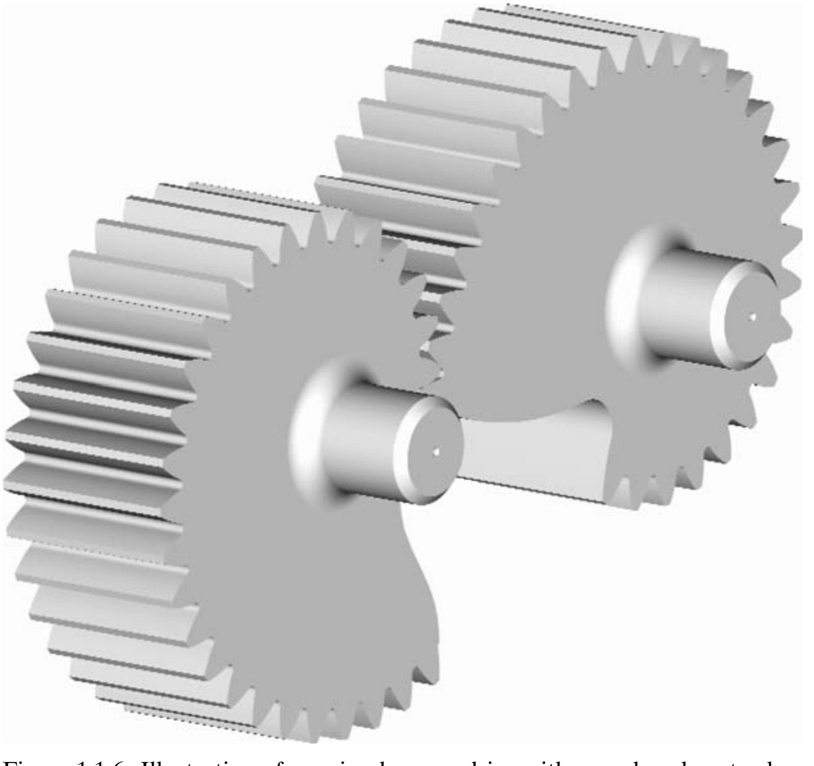

Figure 1.1.6. Illustration of noncircular gear drive with non-closed centrodes.