101 mathematical projects

Brian Bolt & David Hobbs

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101 mathematical projects:
a resource book

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Introduction

In the UK a series of reports on the teaching of mathematics has highlighted the shortcomings of learning mathematical techniques in isolation. The result of this has been a set of national criteria for the teaching of mathematics which emphasises the need for pupils to be taught in such a way that they will be able to use the mathematics they learn. This has been followed by new school-leaving examinations involving coursework projects to promote this aim. To most teachers this means a significant change in what is demanded of them. We wholeheartedly support the change in emphasis in mathematics education but we are also aware of the problems which its implementation will inevitably bring. With this in mind we have drawn on our many years in teacher training to produce this resource book of over a hundred topics which we believe teachers will find invaluable when they introduce coursework.

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Why coursework?

In recent years there has been a growing recognition that pupils should learn mathematics in such a way that they can see its relevance to the world in which they live and be able to use it to gain a better appreciation of that world. Often mathematics has been learned as a set of routines to be carried out blindly in response to stereotyped exam questions. The result of such teaching and learning is that pupils are unable to apply their knowledge outside the standard textbook sums. Further, the motivation for learning becomes largely dependent on getting the ticks corresponding to right answers and has little to do with any intrinsic interest in the subject or whether or not the answer is meaningful.

When mathematics is taught in that way pupils rarely, if ever, have opportunity to ask their own questions. The questions come to them from textbook exercises, workcards or worksheets, or exam papers, and no matter how carefully they have been designed they have come from an external source beyond pupils' control.

The way in which pupils are ultimately assessed has a very strong influence on the way in which mathematics is taught. As long as the school-leaving assessment is based on timed written papers with a large number of questions to be answered then little will change. Fortunately for the future of mathematics in the UK this has now been recognised.

The Cockcroft Report: *Mathematics Counts* (HMSO 1982) spelt out, among other things, that mathematics teaching at all levels should include: discussion, practical work, investigational work, problem solving, and application of mathematics to everyday situations.

The HM Inspectorate developed the ideas inherent in the Cockcroft Report in their discussion document: *Mathematics from 5 to 16* (HMSO 1985) where they spelt out a set of aims and objectives including the need to develop independent thinking:

> There is a danger that mathematics might be made to appear to pupils to consist mainly of answering set questions, often of a trivial nature, to which the answers are already known and printed in the answer book! But *pupils will have developed well mathematically when they are asking and answering their own questions* . . . why? . . . how? . . . what does that mean? . . . is there a better way? . . . what would happen if I changed that? . . . does the order matter? . . .

Parallel to these reports has come the development of National Criteria for secondary mathematics, and a new examination, the General Certificate for Secondary Education (GCSE) to be taken at 16+.

In the introduction to the National Criteria for mathematics it states that any scheme of assessment must:

(a) assess not only the performance of skills and techniques but also pupils' understanding of mathematical processes, their ability to make use of these processes in the solution of problems and their ability to reason mathematically;

(b) encourage and support the provision of courses which will enable pupils to develop their knowledge and understanding of mathematics to the full extent of their capabilities, to have experience of mathematics as a means of solving practical problems and to develop confidence in their use of mathematics.

It has been appreciated that these aims cannot all be met by written papers but are best met by an element of coursework done in the two

years prior to the examination. This coursework element cannot be obtained by accumulating pieces of homework or tests but is to consist of practical work and investigations which require independence and initiative on the part of the individual. The term 'extended piece of work' is used and this has the valuable ingredient of a task performed over a significant period of time, a feature lacking when work is set on one day and collected in on the next.

Suggested activities include

(a) problems or tasks, which because they are unfamiliar, give opportunity to develop initiative and flexibility and so encourage a spirit of enquiry;
(b) tasks in which a variety of strategies and skills can be used;
(c) problems and surveys in which information has to be gathered and inferences have to be made;
(d) situations which can be investigated, with opportunities for strategies such as trial and error and searching for pattern;
(e) extended pieces of work which enable a pupil to investigate a topic or problem at length;
(f) opportunities for pupils to generate their own investigative activities.

However, as is pointed out, the ability to carry out these activities loses much unless the pupils can communicate their findings to others. It follows that pupils need to develop the ability to describe what they have achieved using words, diagrams, graphs or formulae as appropriate. And last but not least they are to be encouraged to talk about their findings.

This coursework element seems a daunting task to teachers whose main concern has been to prepare pupils for written examinations. Many find themselves having to teach in ways which they have not themselves experienced when pupils, so they have no model to fall back on. Inservice courses are helpful as pump priming exercises but in the end a teacher needs a source of ideas presented in a form which can be readily used with pupils. It was with this in mind that this book has been written. It contains many topics, giving ways in which they may be developed, and the kinds of questions which pupils can be encouraged to ask and seek answers to. Many of the topics can be developed in a variety of ways and the depth and width of any project based on them will depend on the ability of the pupil concerned and the time scale envisaged for the project to be completed. The range of topics included has been chosen to cater for a variety of interests and to cover a wide range of concepts and skills.

Teachers often look for a situation to illustrate or motivate an interest in a piece of mathematics which they want to introduce. This is still relevant, but in doing the coursework element of GCSE it should be the intrinsic interest and relevance of the problem which takes precedence, not the mathematics.

In writing this book we have concentrated on projects which have links with the real world to emphasise the relevance of mathematics to a better understanding of our environment. We have consciously omitted the pure mathematics investigations such as those involving number patterns or shape, unless they have tangible links with real problems, as these are already well resourced. We have thus grouped the project topics into themes such as measurement, sport, the home, transport and technology rather than into topics such as statistics, scale drawing, and algebra. The situation should determine what aspects of mathematics are used and in most cases several techniques will be involved. For example a project based on sport may look at the characteristics of a bouncing ball and involve designing an experiment, measuring, graphical representation and a theoretical model *or* it may make a study of the effect of different scoring systems on the outcome of sporting competitions and suggest possible alternatives with an analysis of the likely outcome of their implementation.

The level of mathematics in a project will often be quite low. In practice much of the mathematical content of a project is likely to consist of activities such as: estimating, measuring, collecting and recording data, drawing graphs, scale drawings, and straightforward arithmetic. It is in the planning of the project, the design of an experiment, the search for information, the questions asked, the conclusions formed, and the communication of the findings where this aspect of the mathematics course differs from the traditional curriculum.

The research of the Assessment of Performance Unit (APU) which regularly monitors pupils' performance in mathematics also assesses their attitude to the subject. It shows that the single most significant factor in creating a positive attitude to mathematics is a pupil's perception of the usefulness of the subject. This is true whether or not a child finds the subject easy, and it becomes more marked as a child grows older. The traditional secondary school mathematics curriculum has always attempted to show mathematics to be useful, but the questions were often contrived to try to use the mathematics being taught or on topics like 'stocks and shares' which could hardly seem relevant to a sixteen-year-old. The questions and examples were imposed from outside. Now there is the opportunity, which teachers must rapidly acknowledge, of allowing pupils to tackle their own problems and in so doing grow in independence and confidence and make the subject their own.

The essence of this book is in the project outlines but some guidelines are given towards starting project work and how to assess the results. Further we give a list of useful resource books and materials. By the time teachers have been involved in coursework for a few years they will realise how limitless is the list of starting points for pupils' coursework. Meanwhile we believe we have put together a wide ranging set of starting points which will give confidence to teachers embarking on this work and add to the possibilities of those with some experience.

Introducing coursework in the classroom

The examining boards' coursework requirements normally refer to assignments carried out in the two years leading up to the final examination date. However, it would be a grave mistake to delay the start of project work to this stage. Many pupils will have been involved in project work in their primary schools, so it would be advantageous to see project work introduced in the first year of secondary school and to be an ever-present element of the mathematics course.

Because the introduction of project work has been initiated, in many cases, by the requirements of an examination, there is often an unhealthy concern with assessment and this tends to dominate teachers' discussions. This is unfortunate. Projects should first and foremost be about getting pupils involved as independent thinkers, asking questions, making and testing hypotheses, collecting data, forming conclusions, and communicating their findings. The emphasis on assessment leads to a concern with making sure that work is only that of an individual when it would be far better to encourage cooperative effort and team work. With this last point in mind we would suggest that, in many cases, project work should be planned and carried out by teams of pupils. This makes sense for example in measuring activities or traffic surveys, and in many practical situations. In fact the discussion between the members of a team and their joint planning is an invaluable part of this aspect of the course.

Take, for example, the problems of car parking. There are many aspects of this, and following a class discussion to identify specific problems, teams of three or four could be formed to pursue them in more detail. The teams would be expected to do what was necessary to analyse their problems and then present their findings to the rest of the class. This presentation could be in the form of a written or oral report, or a wall display or a model or using a micro. One team, for example, might make a study of a local car park, another look at street parking and another at the possibilities of forming a car park from the school playground for a special function. Such a topic will involve measurement, data collection, surveying, graphs, planning and decision making to name just a few of the skills, and if it can be linked to a real problem so much the better.

Some projects, such as a statistical analysis of the contents of different newspapers, can easily be carried out by individuals but would be more rewarding if done by groups of pupils because of the inevitable discussion which will arise and the saving in time on what could become a repetitive and possibly boring task for an individual. In a group there will always be someone who does more than their fair share and someone who takes a back seat, but that is life, and learning how to work as a team is as important a skill to acquire as the insights gained into using mathematics. Pupils too are often more ready to discuss and learn from each other than from the teacher.

The new Scottish Standard Grade has incorporated practical investigations and makes the point of the desirability of working together to develop social and personal qualities. It also includes the following relevant paragraph:

> Working co-operatively with others is a powerful way of tackling problems. Moreover, the exchange of ideas through discussion is an essential part of learning. Activities are required to develop the ability to work with others towards a common goal, or for a common purpose.

One problem which arises from group work is how much each individual is expected to record. Is a group project a shared experience which only lives on in the memory of the individuals or does each person write up a complete report? Projects must be viewed positively by both pupils and teachers and must not become a burden. On the other hand, if detailed reports are not produced the activities will become rather pointless. A compromise must be found, and one solution is for a detailed write-up to be produced by a group which is available for all to see, together with skeleton reports with the main results and conclusions which each pupil can keep in their coursework file.

It is not easy to generalise about how to set about doing a project. In the beginning it will help if the projects are carefully structured and fairly limited in scope. They may well be closely linked to the mathematics syllabus being taught at the time, but pupils will be able to show more independence if the projects in which they are involved depend more on using mathematics in which they are already reasonably competent. Later the projects can be much more open-ended and may in fact be proposed by the pupils themselves. The choice of topic for a project will influence to some extent the stages involved but the following framework is offered as a guideline:

1 Interpreting the task

Having chosen a topic the first stage is to come to terms with what might be involved. What kinds of question can be asked? What information is given or is readily available? What can be measured? What data can be collected? Who might have relevant information? What has the library to offer?

2 Selecting a line of attack

Having taken in the possibilities of the situation some decision has to be made as to which particular aspect attention should be focussed on. Pupils may initially be tempted into trying to be too comprehensive in their approach and will need guidance to narrow down and define a problem which is sufficiently limited for them to achieve a result before they lose interest.

3 Planning and implementation

Having decided on a strategy the need is then to implement it. What information is required and how will it be obtained? How will measurements be made or data collected and how will it be recorded? Who is available to help and when will be a suitable time to carry out any survey? What equipment will be needed and from where can it be obtained?

At this stage it could be helpful to write down, possibly in the form of a flow diagram, what needs to be done and who will do it, before any action takes place.

4 Recording and processing

When the data is collected it needs to be recorded in a meaningful form. This might be, for example, in a table, a bar chart, a pie chart, or a scatter diagram. The processing may involve drawing graphs, calculating means, making models or computing. Questions may arise about relationships between sets of data, and hypotheses can be proposed and tested.

5 Extension

In the process of doing a project it is quite likely that further or related questions propose themselves which could be pursued or presented as problems requiring further research.

6 Presentation

When writing up a report it is helpful for the pupils to see themselves as consultants writing a document for a third party, rather like a surveyor might write a report on a house for a

possible purchaser. The result of a project may end up as a scale drawing, say of a proposed house extension, or a series of models to demonstrate how shapes fill space or how folding push-chairs operate. Alternatively a wall display with pictograms and pie charts of a statistical survey or a display of patchwork patterns with an analysis of the unit of design may be more appropriate. But presentations could well include expositions by individuals or teams making use of the blackboard, OHP, models or any form of visual aid they can devise. Opportunities to communicate their findings in this way with the follow-up questions from their peer group would take time but could be an invaluable part of the exercise.

The teacher's role in project work is all important. To start with it is probably easier for the teacher to give the same project to all the pupils, so that setting the scene has only to be done once, and for the teacher to keep close control over its development. Take, for example, project 10 based on bouncing balls. After an initial discussion with the class about the wide range of balls used in different sports pupils should become aware of the need to find a way of measuring how well a ball bounces, and the need for manufacturers to produce balls with a consistent bounce for each sport. From this discussion should emerge the idea of dropping a ball from a known height and seeing to what height it bounces as a suitable way of measuring a ball's bounciness. The teacher will need to have available a number and range of balls together with measuring tapes or metre rules so that the class can divide up into groups of three to investigate the characteristics of balls such as:

- How does the bounce of a ball change with the height from which it is dropped?
- How does the bounce of a ball change with the surface onto which it is dropped?
- Which bounces best, a marble, a golf ball, or a netball?

A double period should be sufficient to get this project off the ground and it should end with a feedback session where each group briefly reports on their findings to date. This could be followed by homework where each pupil (a) writes about why manufacturers need to be able to measure the bounce of a ball and (b) describes the experiment they have carried out together with their results and conclusions.

The project could stop at this point, but much more is achieved if at least another double period is given over to it when groups could (a) try to answer for themselves the questions previously tackled by the other groups and (b) look at other related problems such as the lengths of consecutive bounces, the bounce of a ball off a racket or the effect of temperature.

Following this it would be helpful for the pupils if the teacher constructs a set of notes on the board, from class discussion, which sets out the main questions investigated and the conclusions found together with further questions yet to be answered.

At this stage the assessment takes a back seat, but from joint efforts of this kind pupils will develop an understanding of how to approach and write up a project, so that from being largely teacher led the move can be gradually made over the years to the projects being almost entirely dependent on individual pupils. A class may, for example, be given a choice of doing a project on the postal service, or the milk supply, or waste disposal, and initially be given a free hand as to what to do, only being offered advice or possible approaches as the need arises. This kind of project will necessarily take place largely in the pupils' own time for it will require the search for facts outside of school. In this case a time limit should be given, say three weeks, in which no other mathematics homework is set, and opportunities given in class throughout this time to talk with individuals about their progress and to give encouragement and suggest references. Pupils should be encouraged to discuss their projects with each other and share findings but, in the end, which

aspects of the situation a pupil investigates and the way it is written up will be very much the work of an individual.

Only in the final years when the projects are to be assessed as part of an external examination is it necessary to ensure that the work written up and assessed is the unaided work of the pupil concerned. But 'unaided' is not easy to define, for if a pupil shows enough initiative to seek out people who are knowledgeable and can suggest ideas to improve their project this should be applauded. What we are really looking for is that a pupil has come to terms with the project and the write-up is their own.

As pupils become more experienced in pursuing projects the teacher can keep a low profile. Having initiated a project the pupils should try to ask the questions and provide the answers. Teachers should encourage, give advice, and make suggestions but they need to try above all to leave the initiative and responsibility for their projects with the pupils. Our experience is that when pupils are given this responsibility they often surprise themselves, let alone their teachers, with what they achieve. But don't expect too much too soon! In the early stages the projects should be structured by the teacher after discussion with the class and gradually the pupils can be given more independence.

The best way to get a pupil involved is often to start with a pupil's interest or hobby whether it is stamp collecting, cycling or pop music. In this way they will approach project work with confidence for they will have something to contribute and often be in the position of being more knowledgeable than you, the teacher. Then it will be your role to help them to develop a worthwhile project around their chosen area by asking questions as an interested outsider. The only danger in this approach is that you may end up with an interesting account of a person's hobby but with little or no mathematics. So be warned, and try to point your pupils towards some aspect of their hobbies which can be quantified.

Projects are an excellent vehicle for cooperative work, they also give opportunity for practising basic skills in a meaningful context, but in selecting projects it is well to remember the statement emphasised in the Cockcroft Report:

> We believe it should be a fundamental principle that no topic should be included unless it can be developed sufficiently for it to be applied in ways in which the pupil can understand.

This statement refers to mathematical topics but it clearly expresses the philosophy which we believe should permeate the teaching of mathematics, and the projects will be the medium through which most pupils will be able to demonstrate their understanding of mathematics.

Assessment

Not only does the inclusion of coursework in mathematics syllabuses bring a different emphasis to the *learning* of mathematics, giving students opportunity to investigate and apply mathematics themselves, it also requires a different style of *assessment*.

In mathematics examinations the mark schemes have always been carefully laid down and marking has been as objective as possible. It is therefore not easy for mathematics teachers to adapt to a less precise style, although it should be remembered that marking in other fields such as Art, English and History has always involved a certain amount of subjectivity.

Clearly it is necessary at a national level to provide assessment criteria for coursework which can command respect. The dilemma is that over-prescription of the coursework content and of the assessment criteria will prevent the aims of the coursework from being realised. As the Northern Examining Association says in its GCSE syllabus (1988):

> Coursework is envisaged as enhancing both the curriculum and the assessment. It is seen as a means of widening the scope of the examination and of providing an opportunity for the assessment of mathematical abilities which cannot easily be assessed by means of written papers. The aim is one of making what is important measurable rather than of making what is measurable important. The incorporation of a coursework element in the GCSE Mathematics examination is seen, therefore, as being concerned with pedagogy at least as much as it is with assessment.

The development of criteria

For teachers who do not have much experience of coursework we suggest that they begin with younger children where it is not necessary to give such a high priority to assessment, rather than at the fifteen- and sixteen-year-old stage. At first it might be advisable to begin with short activities before launching out on some of the more extended projects. For example, an activity accessible to eleven- and twelve-year-olds is to design a book of stamps (see project 60). This could begin with a discussion about points such as:

- the cost of the book (50p, £1, £2, £5?);
- useful values of stamps to be included (based on current first and second class postage rates);
- size of the book (number of stamps per page? number of pages?)

The possibilities for one particular cost could then be analysed by discussion with the whole class. Pupils could then try it out for another cost, working in small groups or for homework. A comparison with the books produced by the Post Office could be made and some market research could take place to find which of various possibilities was the most popular. The results could be written up as a report or as a wall display.

As experience is gained it could be that with eleven- to fourteen-year-olds one project is carried out each term, occupying the lessons for one or two weeks, with the children working, where appropriate, in groups. The outcome would be a presentation of some form: a display of models, wall charts, written booklets, etc. possibly accompanied by a verbal account. The teacher could then initiate a discussion about the projects bringing out points such as:

- Did the group members plan their work carefully?
- Were they correct in what they did?
- Did they present their findings in a clear way?

The achievements of each group would thus be made public, aiming for a mature attitude of help and cooperation. Through these discussions pupils could come to appreciate the standards to aim for and the main criteria on which assessment could be based. As the pupils gain experience they could assess the work of other groups on agreed criteria using, say, a five-point scale. By the fourth and fifth years students should then be capable of conducting projects on an individual basis and should appreciate how they will be assessed.

Some guidelines

The following guidelines are offered for assessment of projects:

1 In looking at a completed project the most obvious feature is the *presentation*:
- Does it communicate?
- Is it clearly expressed?
- Are diagrams, tables, models, etc. clear?
- Has it been carefully put together?

2 A more detailed study of the project involves consideration of its *content*:
- Have relevant questions been posed?
- Has appropriate information been obtained and used?
- Have appropriate mathematical ideas been used?
- Is the mathematics correct?
- Have conclusions been drawn?
- Have extensions been undertaken?

3 In some cases it might be appropriate to give credit for the *doing* of the project:
- Was it initiated by the pupil?
- Was teacher support needed?
- Did the pupils develop their own strategy?
- Is there evidence of personal initiative?

Major categories such as these could be assessed on a five-point numerical scale, say, and the results combined, with suitable weightings, to give an overall assessment. Different weightings might be appropriate depending on the ability level of the children. Care is needed in matching projects to pupils especially when they have freedom to choose their own projects. For less able children the projects need to be within their capabilities: project 30, 'Decorating and furnishing a room', offers possibilities for such children. For able children the projects must have potential for involving mathematics at a suitably high level and this should be looked for in the assessment scheme: see, for example, project 79, 'Packing', and project 89, 'Crystals' (second part) where the demands on spatial thinking are high. In some cases it might be that a project can be developed at various levels: project 23, 'Designing games of chance', can be taken at a simple level or extended to games which require careful analysis using probabilistic ideas.

In conclusion, we would like to emphasise again that assessment must be the servant of the curriculum and that what is taught should not be tailored to those aspects of the curriculum which can be measured easily. We would therefore wish to encourage teachers to experiment and, where examining boards allow, to produce their own style of coursework and appropriate assessment. Also, we hope that there would not be a dichotomy between projects and other forms of teaching. Rather, we would hope that an atmosphere of discussion, investigation and problem solving, as we have tried to indicate in the projects, would pervade the teaching of all of the mathematics.

The projects

The projects outlined on the following pages have been classified under a number of headings in order to give structure to the book. Some of these headings correspond to areas specified by the GCSE examination groups.

The projects have deliberately been chosen on a great variety of topics. Clearly not all the examples will appeal to everyone. It is certainly not intended that pupils should work through them systematically. We would like to encourage recognition of the fact that pupils are different and that the work they do in mathematics should match their abilities and interests. Some of the suggestions are at quite a low level while others involve difficult mathematical ideas and are only suitable for the most able students.

In some examples we have tried to encourage an across-the-curriculum approach. For example, there are links with subjects such as Art, Biology, Chemistry, CDT, Geography, Music and Physics. This allows children whose main interest is in some other school subject to build on it in their mathematics lessons. Advice from teachers of these other subjects might be useful; indeed, there could be opportunity for joint projects.

Also the project suggestions have not been written in a uniform style. In some cases we have given mathematical background where it might be unfamiliar; in others we have been briefer. Where possible we have given references, some of which are directly accessible by pupils, and some are at teacher level.

It should be emphasised that the projects are not prescriptive. We have tried to suggest some possible starting points which we hope will spark off other lines of inquiry. Above all, it is the flavour of a project-based approach which we would wish to convey.

List of project topics

35 In the garden
36 Where has all the electricity gone?
37 Energy conservation

Budgeting

38 The cost of keeping a pet
39 The cost of a wedding
40 The real cost of sport
41 Buying or renting a TV
42 A holiday abroad
43 The cost of running ballet/driving/riding schools
44 The cost of running a farm
45 Financial arithmetic

History

46 Numbers and devices for calculation
47 The history of π
48 Pythagoras' theorem
49 Calculating prodigies

Transport

50 Traffic
51 Public transport
52 The flow of traffic around a roundabout
53 Traffic lights
54 Stopping distances
55 Car parking
56 Buying and running your own transport
57 Canals and waterways

Public services

58 The water supply
59 The milk supply
60 The postal service
61 Telephone charges
62 Waste disposal

Technology

63 Triangular frameworks
64 Four-bar linkages
65 Parabolic reflectors
66 How effective is a teacosy?
67 Cycle design
68 Cranes
69 Rollers and rolling
70 Transmitting rotary motion
71 Triangles with muscle

Space

72 Paper sizes and envelopes
73 Measuring inaccessible objects
74 Surveying ancient monuments
75 Paper folding
76 Spirals
77 Patchwork patterns
78 Space filling
79 Packing
80 Cones
81 Three-dimensional representation
82 Three-dimensional surfaces
83 Curves from straight lines

Links with other subjects

84 Mathematics in biology
85 Making maps
86 Mathematics in geography
87 Music and mathematics
88 Photography
89 Crystals

Random number simulations

90 Random numbers
91 Simulating movement
92 Simulating the lifetime of an electrical device
93 Queues

Miscellaneous

94 Letter counts
95 Comparing newspapers
96 Sorting by computer
97 Weighted networks
98 Codes
99 Computer codes
100 Maximising capacity
101 The school