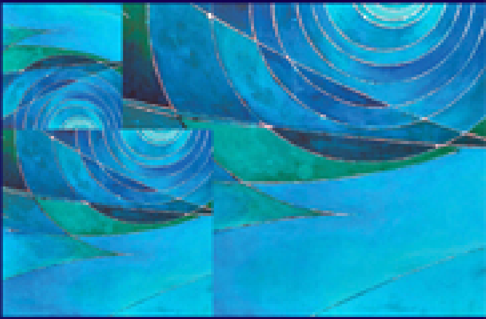
Introduction to

Elementary Particle Physics



Alessandro Bettini

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INTRODUCTION TO ELEMENTARY PARTICLE PHYSICS

The Standard Model is the theory of the elementary building blocks of matter and of their forces. It is the most comprehensive physical theory ever developed, and has been experimentally tested with high accuracy.

This textbook conveys the basic elements of the Standard Model using elementary concepts, without theoretical rigour. While most texts on this subject emphasise theoretical aspects, this textbook contains examples of basic experiments, before going into the theory. This allows readers to see how measurements and theory interplay in the development of physics. The author examines leptons, hadrons and quarks, before presenting the dynamics and surprising properties of the charges of the different forces. The textbook concludes with a brief discussion on the recent discoveries in physics beyond the Standard Model, and its connections with cosmology.

Quantitative examples are given throughout the book, and the reader is guided through the necessary calculations. Each chapter ends in exercises so readers can test their understanding of the material. Solutions to some problems are included in the book, and complete solutions are available to instructors at www.cambridge.org/9780521880213. This textbook is suitable for advanced undergraduate students and graduate students.

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Preface

This book is mainly intended to be a presentation of subnuclear physics, at an introductory level, for undergraduate physics students, not necessarily for those specialising in the field. The reader is assumed to have already taken, at an introductory level, nuclear physics, special relativity and quantum mechanics, including the Dirac equation. Knowledge of angular momentum, its composition rules and the underlying group theoretical concepts is also assumed at a working level. No prior knowledge of elementary particles or of quantum field theories is assumed.

The Standard Model is the theory of the fundamental constituents of matter and of the fundamental interactions (excluding gravitation). A deep understanding of the 'gauge' quantum field theories that are the theoretical building blocks of this model requires skills that the readers are not assumed to have. However, I believe it to be possible to convey the basic physics elements and their beauty even at an elementary level. 'Elementary' means that only knowledge of elementary concepts (in relativistic quantum mechanics) is assumed. However it does not mean a superficial discussion. In particular, I have tried not to cut corners and I have avoided hiding difficulties, whenever was the case. I have included only well-established elements with the exception of the final chapter, in which I survey the main challenges of the present experimental frontier.

The text is designed to contain the material that may be accommodated in a typical undergraduate course. This condition forces the author to hard, and sometimes difficult, choices. The chapters are ordered in logical sequence. However, for a short course, a number of sections, or even chapters, can be left out. This is achieved at the price of a few repetitions. In particular, the treatments of oscillation and of the $\mathcal{CP}$ violation phenomena are given in an increasingly advanced way, first for the K mesons, then for the B mesons and finally for neutrinos.

The majority of the texts on elementary particles place special emphasis on theoretical aspects. However, physics is an experimental science and only experiment can decide which of the possible theoretical schemes has been chosen by Nature. Moreover, the progress of our understanding is often due to the discovery of unexpected phenomena. I have tried to select examples of basic experiments first, and then to go on to the theoretical picture.

A direct approach to the subject would start from leptons and quarks and their interactions and explain the properties of hadrons as consequences. A historical approach would also discuss the development of ideas. The former is shorter, but is lacking in depth. I tried to arrive at a balance between the two views.

The necessary experimental and theoretical tools are presented in the first chapter. From my experience, students have a sufficient knowledge of special relativity, but need practical exercise in the use of relativistic invariants and Lorentz transformations. In the first chapter I also include a summary of the artificial and natural sources of high-energy particles and of detectors. This survey is far from being complete and is limited to what is needed for the understanding of the experiments described in the following chapters.

The elementary fermions fall into two categories: the leptons, which can be found free, and the quarks, which always live inside the hadrons. Hadrons are non-elementary, compound structures, rather like nuclei. Three chapters are dedicated to the ground-level hadrons (the *S* wave nonets of pseudoscalar and vector mesons and the *S* wave octet and decimet of baryons), to their symmetries and to the measurement of their quantum numbers (over a few examples). The approach is partly historical.

There is a fundamental difference between hadrons on the one hand and atoms and nuclei on the other. While the electrons in atoms and nucleons in nuclei move at non-relativistic speeds, the quarks in the nucleons move almost at the speed of light. Actually, their rest energies are much smaller than their total energies. Subnuclear physics is fundamentally relativistic quantum mechanics.

The mass of a system can be measured if it is free from external interaction. Since the quarks are never free, for them the concept of mass must be extended. This can be done in a logically consistent way only within quantum chromodynamics (QCD).

The discoveries of an ever-increasing number of hadrons led to a confused situation at the beginning of the 1960s. The development of the quark model suddenly put hadronic spectroscopy in order in 1964. An attempt was subsequently made to develop the model further to explain the hadron mass spectrum. In this programme the largest fraction of the hadron mass was assumed to be due to the quark masses. Quarks were supposed to move slowly, at non-relativistic speeds inside the hadrons. This model, which was historically important in the development of the correct description of hadronic dynamics, is not satisfactory however. Consequently, I will limit the use of the quark model to classification.

The second part of the book is dedicated to the fundamental interactions and the Standard Model. The approach is substantially more direct. The most important experiments that prove the crucial aspects of the theory are discussed in some detail. I try to explain at an elementary level the space-time and gauge structure of the different types of 'charge'. I have included a discussion of the colour factors, giving examples of their attractive or repulsive character. I try to give some hint of the origin of hadron masses and of the nature of vacuum. In the weak interaction chapters the chiralities of the fermions and their weak couplings are discussed. The Higgs mechanism, the theoretical mechanism that gives rise to the masses of the particles, has not yet been tested experimentally. This will be done at the new high-energy large-hadron collider, LHC, now becoming operational at CERN. I shall only give a few hints about this frontier challenge.

In the final chapter I touch on the physics that has been discovered beyond the Standard Model. Actually, neutrino mixing, masses, oscillations and flavour transitions in matter make a beautiful set of phenomena that can be properly described at an elementary level, namely using only the basic concepts of quantum mechanics. Other clues to the physics beyond the Standard Model are already before our eyes. They are due mainly to the increasing interplay between particle physics and cosmology, astrophysics and nuclear physics. The cross fertilisation between these sectors will certainly be one of the main elements of fundamental research over the next few years. I limit the discussion to a few glimpses to give a flavour of this frontier research.

Problems

Numbers in physics are important; the ability to calculate a theoretical prediction on an observable or an experimental resolution is a fundamental characteristic of any physicist. More than 200 numerical examples and problems are presented. The simplest ones are included in the main text in the form of questions. Other problems covering a range of difficulty are given at the end of each chapter (except the last one). In every case the student can arrive at the solution without studying further theoretical material. Physics rather than mathematics is emphasised.

The physical constants and the principal characteristics of the particles are not given explicitly in the text of the problems. The student is expected to look for them in the tables given in the appendices. Solutions for about half of the problems are given at the end of the book.

Appendices

One appendix contains the dates of the main discoveries in particle physics, both experimental and theoretical. It is intended to give a bird's-eye view of the history of the field. However, keep in mind that the choice of the issues is partially arbitrary and that history is always a complex, non-linear phenomenon. Discoveries are seldom due to a single person and never happen instantaneously.

Tables of the Clebsch–Gordan coefficients, the spherical harmonics and the rotation functions in the simplest cases are included in the appendices. Other tables give the main properties of gauge bosons, of leptons, of quarks and of the ground levels of the hadronic spectrum.

The principal source of the data in the tables is the 'Review of Particle Properties' (Yao *et al.* 2006). This 'Review', with its website http://pdg.lbl.gov/, may be very useful to the reader too. It includes not only the complete data on elementary particles, but also short reviews of topics such as tests of the Standard Model, searches for hypothetical particles, particle detectors, probability and statistical methods, etc. However, it should be kept in mind that these 'mini-reviews' are meant to be summaries for the expert and that a different literature is required for a deeper understanding.

Reference material on the Internet

There are several URLs present on the Internet that contain useful material for further reading and data on elementary particles, which are systematically adjourned. The URLs cited in this work were correct at the time of going to press, but the publisher and the author make no undertaking that the citations remain live or accurate or appropriate.

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1

Preliminary notions

1.1 Mass, energy, linear momentum

Elementary particles have generally very high speeds, close to that of light. Therefore, we recall a few simple properties of relativistic kinematics and dynamics in this section and in the next three.

Let us consider two reference frames in rectilinear uniform relative motion $S(t,x,y,z)$ and $S'(t',x',y',z')$. We choose the axes as represented in Fig. 1.1. At a certain moment, which we take as $t' = t = 0$, the origins and the axes coincide. The frame S' moves relative to S with speed $\mathbf{V}$, in the direction of the x-axis.

We introduce the following two dimensionless quantities relative to the motion in S of the origin of S'

$$\boldsymbol{\beta} \equiv \frac{\mathbf{V}}{c} \tag{1.1}$$

and

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{1.2}$$

called the 'Lorentz factor'. An event is defined by the four-vector of the coordinates $(ct,\mathbf{r})$. Its components in the two frames (t,x,y,z) and (t',x',y',z') are linked by the Lorentz transformations (Lorentz 1904, Poincaré 1905)

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct - \beta x). \end{aligned} \tag{1.3}$$

The Lorentz transformations form a group that H. Poincaré, who first recognised this property in 1905, called the Lorentz group. The group contains the parameter c,

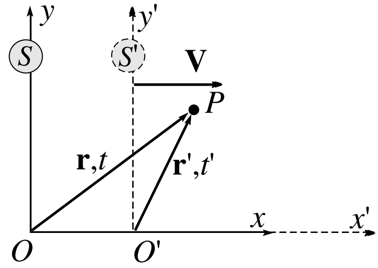

Fig. 1.1. Two reference frames in rectilinear relative motion.

a constant with the dimensions of the velocity. A physical entity moving at speed c in a reference frame moves with the same speed in any other frame. In other words, c is invariant under Lorentz transformations. It is the propagation speed of all the fundamental perturbations: light and gravitational waves (Poincaré 1905).

The same relationships are valid for any four-vector. Of special importance is the energy-momentum vector (E/c, **p**) of a free particle

$$\begin{aligned} p_{x'} &= \gamma\left(p_x - \beta\frac{E}{c}\right) \\ p_{y'} &= p_y \\ p_{z'} &= p_z \\ \frac{E'}{c} &= \gamma\left(\frac{E}{c} - \beta p_x\right). \end{aligned} \tag{1.4}$$

The transformations that give the components in S as functions of those in S', the inverse of (1.3) and (1.4), can be most simply obtained by changing the sign of the speed **V**.

The norm of the energy-momentum vector is, as for all the four-vectors, an invariant; the square of the mass of the system multiplied by the invariant factor c^4

$$m^2c^4 = E^2 - p^2c^2. \tag{1.5}$$

This is a fundamental expression: it is the definition of the mass. It is, we repeat, valid only for a free body but is completely general: for point-like bodies, such as elementary particles, and for composite systems, such as nuclei or atoms, even in the presence of internal forces.

The most general relationship between the linear momentum (we shall call it simply momentum) **p**, the energy E and the speed **v** is

$$\mathbf{p} = \frac{E}{c^2}\mathbf{v} \tag{1.6}$$

which is valid both for bodies with zero and non-zero mass.

For massless particles (1.5) can be written as

$$pc = E. \tag{1.7}$$

The photon mass is exactly zero. Neutrinos have non-zero but extremely small masses in comparison to the other particles. In the kinematic expressions involving neutrinos, their mass can usually be neglected.

If $m \neq 0$ the energy can be written as

$$E = m\gamma c^2 \tag{1.8}$$

and (1.6) takes the equivalent form

$$\mathbf{p} = m\gamma\mathbf{v}. \tag{1.9}$$

We call the reader's attention to the fact that one can find in the literature, and not only in that addressed to the general public, concepts that arose when the theory was not yet well understood and that are useless and misleading. One of these is the 'relativistic mass' that is the product $m\gamma$, and the dependence of mass on velocity. The mass is a Lorentz invariant, independent of the speed; the 'relativistic mass' is simply the energy divided by c^2 and as such the fourth component of a four-vector; this of course if $m \neq 0$, while for $m = 0$ relativistic mass has no meaning at all. Another related term to be avoided is the 'rest mass', namely the 'relativistic mass' at rest, which is simply the mass.

The concept of mass applies, to be precise, only to the stationary states, i.e. to the eigenstates of the free Hamiltonian, just as only monochromatic waves have a well-defined frequency. Even the barely more complicated wave, the dichromatic wave, does not have a well-defined frequency. We shall see that there are two-state quantum systems, such as K^0 and B^0, which are naturally produced in states different from stationary states. For the former states it is not proper to speak of mass and of lifetime. As we shall see, the nucleons, as protons and neutrons are collectively called, are made up of quarks. The quarks are never free and consequently the definition of quark mass presents difficulties, which we shall discuss later.

Example 1.1 Consider a source emitting a photon with energy E_0 in the frame of the source. Take the x-axis along the direction of the photon. What is the energy E of the photon in a frame in which the source moves in the x direction at the speed $\upsilon = \beta c$? Compare with the Doppler effect.

Call S' the frame of the source. Remembering that photon energy and momentum are proportional, we have $p'_x = p' = E_0/c$. The inverse of the last equation in (1.4) gives

$$\frac{E}{c} = \gamma\left(\frac{E_0}{c} + \beta p'_x\right) = \gamma\frac{E_0}{c}(1+\beta)$$

and we have $\frac{E}{E_0} = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}$.

Doppler effect theory tells us that, if a source emits a light wave of frequency ν_0, an observer who sees the source approaching at speed $\upsilon = \beta c$ measures the frequency ν, such that $\frac{\nu}{\nu_0} = \sqrt{\frac{1+\beta}{1-\beta}}$. This is no wonder, in fact quantum mechanics tells us that $E = h\nu$.

1.2 The law of motion of a particle

The 'relativistic' law of motion of a particle was found by Planck in 1906 (Planck 1906). As in Newtonian mechanics, a force $\mathbf{F}$ acting on a particle of mass $m \neq 0$ results in a variation in time of its momentum. Newton's law in the form $\mathbf{F} = d\mathbf{p}/dt$ (the form used by Newton himself) is also valid at high speed, provided the momentum is expressed by Eq. (1.9). The expression $\mathbf{F} = m\mathbf{a}$, used by Einstein in 1905, on the contrary, is wrong. It is convenient to write explicitly

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\gamma\mathbf{a} + m\frac{d\gamma}{dt}\mathbf{v}. \tag{1.10}$$

Taking the derivative, we obtain

$$\begin{aligned} m\frac{d\gamma}{dt}\mathbf{v} &= m\frac{d\left(1-\frac{\upsilon^2}{c^2}\right)^{-1/2}}{dt}\mathbf{v} = -m\frac{1}{2}\left(1-\frac{\upsilon^2}{c^2}\right)^{-3/2}\left(-2\frac{\upsilon}{c^2}a_t\right)\mathbf{v} \\ &= m\gamma^3(\mathbf{a}\cdot\boldsymbol{\beta})\boldsymbol{\beta}. \end{aligned}$$

Hence

$$\mathbf{F} = m\gamma\mathbf{a} + m\gamma^3(\mathbf{a}\cdot\boldsymbol{\beta})\boldsymbol{\beta}. \tag{1.11}$$

We see that the force is the sum of two terms, one parallel to the acceleration and one parallel to the velocity. Therefore, we cannot define any 'mass' as the ratio between acceleration and force. At high speeds, the mass is *not* the inertia to motion.

To solve for the acceleration we take the scalar product of the two members of Eq. (1.11) with $\boldsymbol{\beta}$. We obtain

$$\mathbf{F}\cdot\boldsymbol{\beta} = m\gamma\mathbf{a}\cdot\boldsymbol{\beta} + m\gamma^3\beta^2\mathbf{a}\cdot\boldsymbol{\beta} = m\gamma\left(1+\gamma^2\beta^2\right)\mathbf{a}\cdot\boldsymbol{\beta} = m\gamma^3\mathbf{a}\cdot\boldsymbol{\beta}.$$

Hence

$$\mathbf{a} \cdot \boldsymbol{\beta} = \frac{\mathbf{F} \cdot \boldsymbol{\beta}}{m\gamma^3}$$

and, by substitution into (1.11)

$$\mathbf{F} - (\mathbf{F} \cdot \boldsymbol{\beta})\boldsymbol{\beta} = m\gamma\mathbf{a}.$$

The acceleration is the sum of two terms, one parallel to the force, and one parallel to the speed.

Force and acceleration have the same direction in two cases only: (1) force and velocity are parallel: $\mathbf{F} = m\gamma^3\mathbf{a}$; (2) force and velocity are perpendicular: $\mathbf{F} = m\gamma\mathbf{a}$. Notice that the proportionality constants are different.

In order to have simpler expressions in subnuclear physics the so-called 'natural units' are used. We shall discuss them in Section 1.5, but we anticipate here one definition: without changing the SI unit of time, we define the unit of length in such a way that $c = 1$. In other words, the unit length is the distance the light travels in a second in vacuum, namely 299 792 458 m, a very long distance. With this choice, in particular, mass, energy and momentum have the same physical dimensions. We shall often use as their unit the electronvolt (eV) and its multiples.

1.3 The mass of a system of particles, kinematic invariants

The mass of a system of particles is often called 'invariant mass', but the adjective is useless; the mass is always invariant.

The expression is simple only if the particles of the system do not interact amongst themselves. In this case, for n particles of energies E_i and momenta $\mathbf{p}_i$, the mass is

$$m = \sqrt{E^2 - P^2} = \sqrt{\left(\sum_{i=1}^{n} E_i\right)^2 - \left(\sum_{i=1}^{n} \mathbf{p}_i\right)^2}. \tag{1.12}$$

Consider the square of the mass which we shall indicate by s, obviously an invariant quantity

$$s = E^2 - P^2 = \left(\sum_{i=1}^{n} E_i\right)^2 - \left(\sum_{i=1}^{n} \mathbf{p}_i\right)^2. \tag{1.13}$$

Notice that s cannot be negative

$$s \geq 0. \tag{1.14}$$

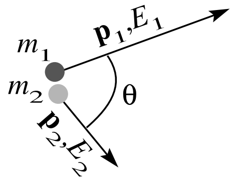

Fig. 1.2. System of two non-interacting particles.

Let us see its expression in the 'centre of mass' (CM) frame that is defined as the reference in which the total momentum is zero. We see immediately that

$$s = \left(\sum_{i=1}^{n} E_i^* \right)^2 \tag{1.15}$$

where E_i^* are the energies in the centre of mass frame. In words, the mass of a system of non-interacting particles is also its energy in the centre of mass frame.

Consider now a system made up of two non-interacting particles. It is the simplest system and also a very important one. Figure 1.2 defines the kinematic variables.

The expression of s is

$$s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = m_1^2 + m_2^2 + 2E_1E_2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 \tag{1.16}$$

and, in terms of the velocity, $\boldsymbol{\beta} = \mathbf{p}/E$

$$s = m_1^2 + m_2^2 + 2E_1E_2(1 - \boldsymbol{\beta}_1 \cdot \boldsymbol{\beta}_2). \tag{1.17}$$

Clearly in this case, and as is also true in general, the mass of a system is not the sum of the masses of its constituents, even if these do not interact. It is also clear from Eq. (1.12) that energy and momentum conservation implies that the mass is a conserved quantity: in a reaction such as a collision or decay, the mass of the initial system is always equal to that of the final system. For the same reason the sum of the masses of the bodies present in the initial state is generally different from the sum of the masses of the final bodies.

Example 1.2 We find the expressions for the mass of the system of two photons of the same energy E, if they move in equal or in different directions.

The energy and the momentum of the photon are equal, because its mass is zero, $p = E$. The total energy $E_{\text{tot}} = 2E$.

If the photons have the same direction then the total momentum is $p_{\text{tot}} = 2E$ and therefore the mass is $m = 0$.

If the velocities of the photons are opposite, $E_{\mathrm{tot}}=2E$, $p_{\mathrm{tot}}=0$, and hence $m=2E$.

In general, if θ is the angle between the velocities, $p_{\mathrm{tot}}^2 = 2p^2 + 2p^2\cos\theta = 2E^2(1+\cos\theta)$ and hence $m^2 = 2E^2(1-\cos\theta)$.

Notice that the system does not contain any matter, but only energy. Contrary to intuition, mass is *not* a measure of the quantity of matter in a body.

Now consider one of the basic processes of subnuclear physics, collisions. In the initial state two particles, a and b, are present, in the final state we may have two particles (not necessarily a and b) or more. Call these c, d, e,... The process is

$$a + b \to c + d + e + \cdots. \tag{1.18}$$

If the final state contains the initial particles, and only them, then the collision is said to be elastic.

$$a + b \to a + b. \tag{1.19}$$

We specify that the excited state of a particle must be considered as a different particle.

The time spent by the particles in the interaction, the collision time, is extremely short and we shall think of it as instantaneous. Therefore, the particles in both the initial and final states can be considered as free.

We shall consider two reference frames, the centre of mass frame already defined above and the laboratory frame (L). The latter is the frame in which, before the collision, one of the particles called the target is at rest, while the other, called the beam, moves against it. Let a be the beam particle, m_a its mass, $\mathbf{p}_a$ its momentum and E_a its energy; let b be the target particle and m_b its mass. Figure 1.3 shows the system in the initial state.

In the laboratory frame, s is given by

$$s = (E_a + m_b)^2 - p_a^2 = m_a^2 + m_b^2 + 2m_bE_a. \tag{1.20}$$

In practice, the energy of the projectile is often, but not always, much larger than both the projectile and the target masses. If this is the case, we can approximate Eq. (1.20) by

$$s \approx 2m_bE_a \qquad (E_a, E_b \gg m_a, m_b). \tag{1.21}$$

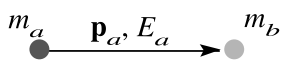

Fig. 1.3. The laboratory frame (L).

We are often interested in producing new types of particles in the collision, and therefore in the energy available for such a process. This is obviously the total energy in the centre of mass, which, as seen in (1.21), grows proportionally to the square root of the beam energy.

Let us now consider the centre of mass frame, in which the two momenta are equal and opposite, as in Figure 1.4. If the energies are much larger than the masses, $E_a^* \gg m_a$ and $E_b^* \gg m_b$, the energies are approximately equal to the momenta: $E_a^* \approx p_a^*$ and $E_b^* \approx p_b^*$, hence equal to each other, and we call them simply E^*. The total energy squared is

$$s \approx (2E^*)^2 \quad \left(E^* \gg m_a, m_b\right). \tag{1.22}$$

We see that the total centre of mass energy is proportional to the energy of the colliding particles. In the centre of mass frame, all the energy is available for the production of new particles, in the laboratory frame only part of it is available, because momentum must be conserved.

Now let us consider a collision with two particles in the final state: the two-body scattering

$$a + b \rightarrow c + d. \tag{1.23}$$

Figure 1.5 shows the initial and final kinematics in the laboratory and in the centre of mass frames. Notice in particular that in the centre of mass frame the final momentum is in general different from the initial momentum; they are equal *only* if the scattering is elastic.

Since s is an invariant it is equal in the two frames; since it is conserved it is equal in the initial and final states. We have generically in any reference frame

$$s = (E_a + E_b)^2 - (\mathbf{p}_a + \mathbf{p}_b)^2 = (E_c + E_d)^2 - (\mathbf{p}_c + \mathbf{p}_d)^2. \tag{1.24}$$

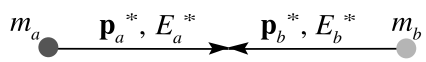

Fig. 1.4. The centre of mass reference frame (CM).

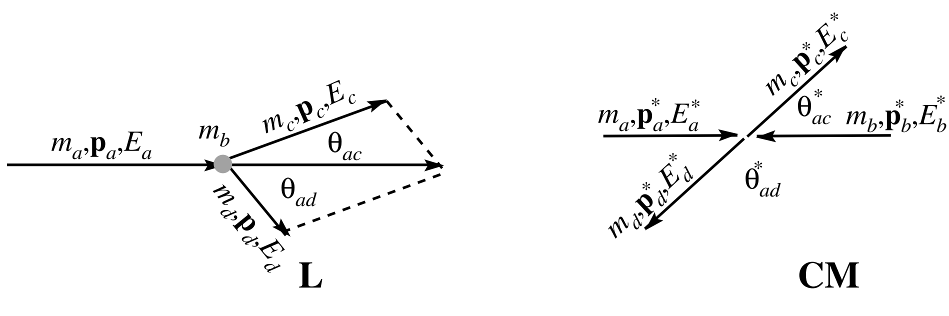

Fig. 1.5. Two-body scattering in the L and CM frames.

These properties are useful to solve a number of kinematic problems, as we shall see later in the 'Problems' section.

In a two-body scattering, there are two other important kinematic variables that have the dimensions of the square of an energy: the *a–c* four-momentum transfer t, and the *a–d* four-momentum transfer u. The first is defined as

$$t \equiv \left(E_c - E_a\right)^2 - \left(\mathbf{p}_c - \mathbf{p}_a\right)^2. \tag{1.25}$$

It is easy to see that the energy and momentum conservation implies

$$t = \left(E_c - E_a\right)^2 - \left(\mathbf{p}_c - \mathbf{p}_a\right)^2 = \left(E_d - E_b\right)^2 - \left(\mathbf{p}_d - \mathbf{p}_b\right)^2. \tag{1.26}$$

In a similar way

$$u \equiv \left(E_d - E_a\right)^2 - \left(\mathbf{p}_d - \mathbf{p}_a\right)^2 - \left(E_c - E_b\right)^2 - \left(\mathbf{p}_c - \mathbf{p}_b\right)^2. \tag{1.27}$$

The three variables are not independent. It is easy to show (see Problems) that

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2. \tag{1.28}$$

Notice finally that

$$t \leq 0 \qquad u \leq 0. \tag{1.29}$$

1.4 Systems of interacting particles

Let us now consider a system of interacting particles. We immediately stress that its total energy is not in general the sum of the energies of the single particles, $E \neq \sum_{i=1}^{n} E_i$, because the field responsible for the interaction itself contains energy. Similarly, the total momentum is not the sum of the momenta of the particles, $\mathbf{P} \neq \sum_{i=1}^{n} \mathbf{p}_i$, because the field contains momentum. In conclusion, Eq. (1.12) does not in general give the mass of the system. We shall restrict ourselves to a few important examples in which the calculation is simple.

Let us first consider a particle moving in an external, given field. This means that we can consider the field independent of the motion of the particle.

Let us start with an atomic electron of charge q_e at the distance r from a nucleus of charge Zq_e. The nucleus has a mass $M_N \gg m_e$, hence it is not disturbed by the electron motion. The electron then moves in a constant potential $\phi = -\frac{1}{4\pi\varepsilon_0}\frac{Zq_e}{r}$. The electron energy (in SI units) is

$$E = \sqrt{m_e^2 c^4 + p^2 c^2} - \frac{1}{4\pi\varepsilon_0}\frac{Zq_e^2}{r} \approx m_e c^2 + \frac{p^2}{2m_e} - \frac{1}{4\pi\varepsilon_0}\frac{Zq_e^2}{r}$$

where, in the last member, we have taken into account that the atomic electron speeds are much smaller than c. The final expression is valid in non-relativistic situations, as is the case in an atom, and it is the non-relativistic expression of the energy, apart from the irrelevant constant $m_e c^2$.

Let us now consider a system composed of an electron and a positron. The positron, as we shall see, is the antiparticle of the electron. It has the same mass and opposite charge. The difference from the hydrogen atom is that there is no longer a fixed centre of force. We must consider not only the two particles but also the electromagnetic field in which they move, which, in turn, depends on their motion. If the energies are high enough, quantum processes happen at an appreciable frequency: the electron and the positron can annihilate each other, producing photons; inversely, a photon of the field can 'materialise' in a positron–electron pair. In these circumstances, we can no longer speak of a potential.

In conclusion, the concept of potential is non-relativistic: we can use it if the speeds are small in comparison to c or, in other words, if energies are much smaller than the masses. It is correct for the electrons in the atoms, to first approximation, but not for the quarks in the nucleons.

Example 1.3 Consider the fundamental level of the hydrogen atom. The energy needed to separate the electron from the proton is $\Delta E = 13.6$ eV. The mass of the atom is smaller than the sum of the masses of its constituents by this quantity, $m_{\mathrm{H}} + \Delta E = m_p + m_e$. The relative mass difference is

$$\frac{m_{\mathrm{H}} - m_p - m_e}{m_{\mathrm{H}}} = \frac{13.6}{9.388 \times 10^8} = 1.4 \times 10^{-8}.$$

This quantity is extremely small, justifying the non-relativistic approximation.

Example 1.4 The processes we have mentioned above, of electron–positron annihilation and pair production, can take place only in the presence of another body. Otherwise, energy and momentum cannot be conserved simultaneously. Let us now consider the following processes:

- $\gamma \to e^+ + e^-$. Let E_+ be the energy and $\mathbf{p}_+$ the momentum of e^+, E_- and $\mathbf{p}_-$ those of e^-. In the initial state $s = 0$; in the final state $s = (E_+ + E_-)^2 - (\mathbf{p}_+ + \mathbf{p}_-)^2 = 2m_e^2 + 2(E_+E_- - p_+p_- \cos\theta) > 2\,m_e^2 > 0$. This reaction cannot occur.
- $e^+ + e^- \to \gamma$. This is just the inverse reaction, it cannot occur either.
- $\gamma + e^- \to e^-$. Let the initial electron be at rest, let E_γ be the energy of the photon, E_f, $\mathbf{p}_f$ the energy and the momentum of the final electron. Initially $s = (E_\gamma + m_e)^2 - p_\gamma^2 = 2m_e E_\gamma + m_e^2$, in the final state $s = E_f^2 - p_f^2 = m_e^2$. Setting

the two expressions equal we obtain $2m_eE_\gamma = 0$, which is false. The same is true for the inverse process $e^- \to e^- + \gamma$. This process happens in the Coulomb field of the nucleus, in which the electron accelerates and radiates a photon. The process is known by the German word bremsstrahlung.

Example 1.5 Macroscopically inelastic collision. Consider two bodies of the same mass m moving initially one against the other with the same speed υ (for example two wax spheres). The two collide and remain attached in a single body of mass M.

The total energy does not vary, but the initial kinetic energy has disappeared. Actually, the rest energy has increased by the same amount. The energy conservation is expressed as $2\gamma mc^2 = Mc^2$. The mass of the composite body is $M > 2m$, but by just a little.

Let us see by how much, as a percentage, for a speed of $\upsilon = 300$ m/s. This is rather high by macroscopic standards, but small compared to c, $\beta = \upsilon/c = 10^{-6}$. Expanding in series: $M = 2\gamma m = \dfrac{2m}{\sqrt{1-\beta^2}} \approx 2m(1 + \frac{1}{2}\beta^2)$. The relative mass difference is: $\dfrac{M-2m}{2m} \approx \frac{1}{2}\beta^2 \approx 10^{-12}$.

It is so small that we cannot measure it directly; we do it indirectly by measuring the increase in temperature with a thermometer.

Example 1.6 Nuclear masses. Let us consider a ^{4}He nucleus, which has a mass of $m_{\mathrm{He}} = 3727.41$ MeV. Recalling that $m_p = 938.27$ MeV and $m_n = 939.57$ MeV, the mass defect is $\Delta E = \left(2m_p + 2m_n\right) - m_{\mathrm{He}} = 28.3\,\mathrm{MeV}$, or, in relative terms, $\dfrac{\Delta E}{m_{\mathrm{He}}} = \dfrac{28.3}{3727.41} = 0.8\%$.

In general, the mass defects in the nuclei are much larger than in the atoms; indeed, they are bound by a much stronger interaction.

1.5 Natural units

In the following, we shall normally use the so-called ‘natural units’ (NU). Actually, we have already started to do so. We shall also use the electronvolt instead of the joule as the unit of energy.

Let us start by giving $\hbar$ and c in useful units:

$$\hbar = 6.58 \times 10^{-16}\,\mathrm{eV\,s}. \tag{1.30}$$

$$c = 3 \times 10^{23}\,\mathrm{fm\,s^{-1}}. \tag{1.31}$$

$$\hbar c = 197\,\text{MeV fm (or GeV am)}. \tag{1.32}$$

As we have already done, we keep the second as unit of time and define the unit of length such that $c = 1$. Therefore, in dimensional equations we shall have $[L] = [T]$.

We now define the unit of mass in such a way as to have $\hbar = 1$. Mass, energy and momentum have the same dimensions: $[M] = [E] = [P] = [L^{-1}]$.

For unit conversions the following relationships are useful:

$$1\,\text{MeV} = 1.52\times10^{21}\ \text{s}^{-1} \qquad 1\ \text{MeV}^{-1} = 197\,\text{fm}$$

$$1\,\text{s} = 3\times10^{23}\ \text{fm} \qquad 1\ \text{s}^{-1} = 6.5\times10^{-16}\ \text{eV} \qquad 1\ \text{ps}^{-1} = 0.65\,\text{meV}$$

$$1\,\text{m} = 5.07\times10^{6}\ \text{eV}^{-1} \qquad 1\,\text{m}^{-1} = 1.97\times10^{-7}\ \text{eV}.$$

The square of the electron charge is related to the fine structure constant α by the relation

$$\frac{q_e^2}{4\pi\varepsilon_0} = \alpha\hbar c \approx 2.3\times10^{-28}\,\text{J m}. \tag{1.33}$$

Being dimensionless, α has the same value in all unit systems (note that, unfortunately, one can still find in the literature the Heaviside–Lorentz units, in which $\varepsilon_0 = \mu_0 = 1$),

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}. \tag{1.34}$$

Notice that the symbol m can mean both the mass and the rest energy mc^2, but remember that the first is Lorentz-invariant, the second is the fourth component of a four-vector. To be complete, the same symbol may also mean the reciprocal of the Compton length times 2π, $\dfrac{2\pi\hbar}{mc}$.

Example 1.7 Measuring the lifetime of the π^0 meson one obtains $\tau_{\pi^0} = 8.4\times10^{-17}$ s; what is its width? Measuring the width of the η meson one obtains $\Gamma_\eta = 1.3\,\text{keV}$; what is its lifetime? We simply use the uncertainty principle:

$$\Gamma_{\pi^0} = \hbar/\tau_{\pi^0} = \left(6.6\times10^{-16}\,\text{eV s}\right)/\left(8.4\times10^{-17}\,\text{s}\right) = 8\,\text{eV}$$

$$\tau_\eta = \hbar/\tau_\eta = \left(6.6\times10^{-16}\,\text{eV s}\right)/(1300\,\text{eV}) = 5\times10^{-19}\,\text{s}.$$

In conclusion, lifetime and width are completely correlated. It is sufficient to measure one of the two. The width of the π^0 particle is too small to be measured, and so we measure its lifetime; vice versa in the case of the η particle.

Example 1.8 Evaluate the Compton wavelength of the proton.

$$\begin{aligned}\lambda_p &= 2\pi/m = (6.28/938)\,\text{MeV}^{-1} = 6.7\times10^{-3}\,\text{MeV}^{-1} \\ &= 6.7\times10^{-3}\times197\,\text{fm} = 1.32\,\text{fm}.\end{aligned}$$

1.6 Collisions and decays

As we have already stated, subnuclear physics deals with two types of processes: collisions and decays. In both cases the transition amplitude is given by the matrix element of the interaction Hamiltonian between final $|f\rangle$ and initial $|i\rangle$ states

$$M_{fi} = \langle f|H_{\text{int}}|i\rangle. \tag{1.35}$$

We shall now recall the basic concepts and relations.

Collisions Consider the collision $a+b\to c+d$. Depending on what we measure, we can define the final state with more or fewer details: we can specify or not specify the directions of c and d, we can specify or not specify their polarisations, we can say that particle c moves in a given solid angle around a certain direction without specifying the rest, etc. In each case, when computing the **cross section** of the observed process we must integrate on the non-observed variables.

Given the two initial particles a and b, we can have different particles in the final state. Each of these processes is called a ‘channel’ and its cross section is called the ‘partial cross section’ of that channel. The sum of all the partial cross sections is the total cross section.

Decays Consider, for example, the three-body decay $a\to b+c+d$: again, the final state can be defined with more or fewer details, depending on what is measured. Here the quantity to compute is the decay rate in the measured final state. Integrating over all the possible kinematic configurations, one obtains the partial decay rate Γ_{bcd}, or partial width, of a into the $b\ c\ d$ channel. The sum of all the partial decay rates is the **total width** of a. The latter, as we have anticipated in Example 1.7, is the reciprocal of the lifetime: $\Gamma = 1/\tau$.

The **branching ratio** of a into $b\ c\ d$ is the ratio $R_{bcd} = \Gamma_{bcd}/\Gamma$.

For both collisions and decays, one calculates the number of interactions per unit time, normalising in the first case to one target particle and one beam particle, in the second case to one decaying particle.

Let us start with the collisions, more specifically with 'fixed target' collisions. There are two elements:

1. The beam, which contains particles of a definite type moving, approximately, in the same direction and with a certain energy spectrum. The beam intensity I_b is the number of incident particles per unit time, the beam flux Φ_b is the intensity per unit normal section.
2. The target, which is a piece of matter. It contains the scattering centres of interest to us, which may be the nuclei, the nucleons, the quarks or the electrons, depending on the case. Let n_t be the number of scattering centres per unit volume and N_t be their total number (if the beam section is smaller than that of the target, N_t is the number of centres in the beam section).

The interaction rate R_i is the number of interactions per unit time (the quantity that we measure). By definition of the cross section σ of the process, we have

$$R_i = \sigma N_t \Phi_b = W N_t \tag{1.36}$$

where W is the rate per particle in the target. To be rigorous, one should consider that the incident flux diminishes with increasing penetration depth in the target, due to the interactions of the beam particles. We shall consider this issue soon. We find N_t by recalling that the number of nucleons in a gram of matter is in all cases, with sufficient accuracy, the Avogadro number N_A. Consequently, if M is the target mass in kg we must multiply by 10^3, obtaining

$$N_{\text{nucleons}} = M(\text{kg})\left(10^3 \text{kg/g}\right) N_A. \tag{1.37}$$

If the targets are nuclei of mass number A

$$N_{\text{nuclei}} = \frac{M(\text{kg})(10^3\text{kg/g})N_A}{A(\text{mol/g})}. \tag{1.38}$$

The cross section has the dimensions of a surface. In nuclear physics one uses as a unit the barn $= 10^{-28}$ m^2. Its order of magnitude is the geometrical section of a nucleus with $A \approx 100$. In subnuclear physics the cross sections are smaller and submultiples are used: mb, μb, pb, etc.

In NU, the following relationships are useful

$$1 \text{ mb} = 2.5\,\text{GeV}^{-2}, \qquad 1\,\text{GeV}^{-2} = 389\,\mu\text{b}. \tag{1.39}$$

Consider a beam of initial intensity I_0 entering a long target of density ρ (kg/m^3). Let z be the distance travelled by the beam in the target, measured from its entrance point. We want to find the beam intensity $I(z)$ as a function of this distance. Consider a generic infinitesimal layer between z and $z + dz$. If dR_i is the total number of interactions per unit time in the layer, the variation of the intensity in crossing the layer is $dI(z) = -dR_i$. If Σ is the normal section of the target, $\Phi_b(z) = I(z)/\Sigma$ is the flux and σ_{tot} is the total cross section, we have

$$dI(z) = -dR_i = -\sigma_{tot}\Phi_b(z)\,dN_t = -\sigma_{tot}\frac{I(z)}{\Sigma}n_t\Sigma\,dz$$

or

$$\frac{dI(z)}{I(z)} = -\sigma_{tot}n_t\,dz.$$

In conclusion, we have

$$I(z) = I_0 e^{-n_t\sigma_{tot}z}. \tag{1.40}$$

The 'absorption length', defined as the distance at which the beam intensity is reduced by the factor $1/e$, is

$$L_{abs} = 1/(n_t\sigma_{tot}). \tag{1.41}$$

Another related quantity is the 'luminosity' $\mathcal{L}$ [$\mathrm{m^{-2}\,s^{-1}}$], often given in [$\mathrm{cm^{-2}\,s^{-1}}$], defined as the number of collisions per unit time and unit cross section

$$\mathcal{L} = R_i/\sigma. \tag{1.42}$$

Let N_b be the number of incident particles per unit time and Σ the beam section; then $N_b = \Phi_b\Sigma$. Equation (1.36) gives

$$\mathcal{L} = \frac{R_i}{\sigma} = \Phi_b N_t = \frac{N_b N_t}{\Sigma}. \tag{1.43}$$

We see that the luminosity is given by the product of the number of incident particles in a second times the number of target particles divided by the beam section. This expression is somewhat misleading because the number of particles in the target seen by the beam depends on its section. We then express the luminosity in terms of the number of target particles per unit volume n_t and in terms of the length l of the target ($N_t = n_t\Sigma l$). Equation (1.43) becomes

$$\mathcal{L} = N_b n_t l = N_b\rho N_A 10^3 l \tag{1.44}$$

where ρ is the target density.

Example 1.9 An accelerator produces a beam of intensity $I = 10^{13}\,\mathrm{s^{-1}}$. The target is made up of liquid hydrogen ($\rho = 60\ \mathrm{kg\ m^{-3}}$) and $l = 10$ cm. Evaluate its luminosity.

$$\mathcal{L} = I\rho 10^3 l N_A = 10^{13}\times 60\times 10^3\times 0.1\times 6\times 10^{23} = 3.6\times 10^{40}\,\mathrm{m^{-2}\,s^{-1}}.$$

We shall now recall a few concepts that should already be known to the reader. We start with the Fermi 'golden rule', which gives the interaction rate W per target particle

$$W = 2\pi |M_{fi}|^2 \rho(E) \tag{1.45}$$

where E is the *total* energy and $\rho(E)$ is the phase-space volume (or simply the phase space) available in the final state.

There are two possible expressions of phase space: the 'non-relativistic' expression used in atomic and nuclear physics, and the 'relativistic' one used in subnuclear physics. Obviously the rates W must be identical, implying that the matrix element M is different in the two cases. In the non-relativistic formalism neither the phase space nor the matrix element are Lorentz-invariant. Both factors are invariant in the relativistic formalism, a fact that makes things simpler.

We recall that in the non-relativistic formalism the probability that a particle i has the position $\mathbf{r}_i$ is given by the square modulus of its wave function, $|\psi\ (\mathbf{r}_i)|^2$. This is normalised by putting its integral over all volume equal to one.

The volume element dV is a scalar in three dimensions, but not in space-time. Under a Lorentz transformation $\mathbf{r} \to \mathbf{r}'$ the volume element changes as $dV \to dV' = \gamma\, dV$. Therefore, the probability density $|\psi(\mathbf{r}_i)|^2$ transforms as $|\psi(\mathbf{r}_i)|^2 \to |\psi'(\mathbf{r}_i)|^2 = |\psi(\mathbf{r}_i)|^2/\gamma$. To have a Lorentz-invariant probability density, we profit from the energy transformation $E \to E' = \gamma E$ and define the probability density as $|(2E)^{-1/2}\psi(\mathbf{r}_i)|^2$ (the factor 2 is due to a historical convention).

The number of phase-space states per unit volume is d^3p_i/h for each particle i in the final state. With n particles in the final state, the volume of the phase space is therefore

$$\rho_n(E) = (2\pi)^4 \int \prod_{i=1}^{n} \frac{d^3 p_i}{(h)^3 2E_i} \delta\left(\sum_{i=1}^{n} E_i - E\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i - \mathbf{P}\right) \tag{1.46}$$

or, in NU (be careful! $\hbar = 1$ implies $h = 2\pi$)

$$\rho_n(E) = (2\pi)^4 \int \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i (2\pi)^3} \delta\left(\sum_{i=1}^{n} E_i - E\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i - \mathbf{P}\right) \tag{1.47}$$

where δ is the Dirac function. Now we consider the collision of two particles, say a and b, resulting in a final state with n particles. We shall give the expression for the cross section.

The cross section is normalised to one incident particle; therefore, we must divide by the incident flux. In the laboratory frame the target particles b are at rest, the beam particles a move with a speed of, say, $\boldsymbol{\beta}_a$. The flux is the number of particles inside a cylinder of unitary base and height $\boldsymbol{\beta}_a$.

Let us consider, more generally, a frame in which particles b also move, with velocity $\boldsymbol{\beta}_b$, that we shall assume parallel to $\boldsymbol{\beta}_a$. The flux of particles b is their number inside a cylinder of unitary base of height $\boldsymbol{\beta}_b$. The total flux is the number of particles in a cylinder of height $\boldsymbol{\beta}_a - \boldsymbol{\beta}_b$ (i.e. the difference between the speeds, which is not, as is often written, the relative speed). If E_a and E_b are the initial energies the normalisation factors of the initial particles are $1/(2E_a)$ and $1/(2E_b)$. It is easy to show, but we shall only give the result, that the cross section is

$$\sigma = \frac{1}{2E_a 2E_b |\boldsymbol{\beta}_a - \boldsymbol{\beta}_b|} \int |M_{fi}|^2 (2\pi)^4 \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \times \delta\left(\sum_{i=1}^{n} E_i - E\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i - \mathbf{P}\right). \tag{1.48}$$

The case of a decay is simpler, because in the initial state there is only one particle of energy E. The probability of transition per unit time to the final state f of n particles is

$$\Gamma_{if} = \frac{1}{2E} \int |M_{fi}|^2 (2\pi)^4 \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \delta\left(\sum_{i=1}^{n} E_i - E\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i - \mathbf{P}\right). \tag{1.49}$$

With these expressions, we can calculate the measurable quantities, cross sections and decay rates, once the matrix elements are known. The Standard Model gives the rules to evaluate all the matrix elements in terms of a set of constants. Even if we do not have the theoretical instruments for such calculations, we shall be able to understand the physical essence of the principal predictions of the model and to study their experimental verification.

Now let us consider an important case, the **two-body phase space**. Let c and d be the two final-state particles of a collision or decay. We choose the centre of mass frame, in which calculations are easiest. Let E_c and E_d be the energies of the two particles, $E = E_c + E_d$ the total energy, and $\mathbf{p}_f = \mathbf{p}_c = -\mathbf{p}_d$ the momentum. We must evaluate the integral

$$\int |M_{fi}|^2 \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_d}{(2\pi)^3 2E_d} (2\pi)^4 \delta(E_c + E_d - E) \delta^3(\mathbf{p}_c + \mathbf{p}_d).$$

Having the energies and the absolute values of the momenta of the final particles fixed, the matrix element can depend only on the angles. Consider the phase-space integral

$$\rho_2 = \int \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_d}{(2\pi)^3 2E_d} (2\pi)^4 \delta(E_c + E_d - E) \delta^3(\mathbf{p}_c + \mathbf{p}_d).$$

Integrating over d^3p_d we obtain

$$\rho_2 = \frac{1}{(4\pi)^2}\int \frac{d^3p_c}{E_c E_d(p_c)}\delta(E_c + E_d(p_c) - E)$$
$$= \frac{1}{(4\pi)^2}\int \frac{p_f^2\, dp_f\, d\Omega_f}{E_c E_d(p_f)}\delta\big(E_c + E_d(p_f) - E\big).$$

Using the remaining δ-function we obtain straightforwardly

$$\frac{1}{(4\pi)^2}\frac{p_f^2}{E_c E_d(p_f)}\frac{d\,p_f}{d\big(E_c + E_d(p_f)\big)}d\Omega_f = \frac{1}{(4\pi)^2}\frac{p_f^2}{E_c E_d(p_f)}\frac{1}{\dfrac{d}{dp_f}\big(E_c + E_d(p_f)\big)}d\Omega_f.$$

But $\dfrac{dE_c}{dp_f} = \dfrac{p_f}{E_c}$ and $\dfrac{dE_d}{dp_f} = \dfrac{p_f}{E_d}$, hence $\dfrac{1}{(4\pi)^2}\dfrac{p_f^2}{E_c E_d}\dfrac{1}{\dfrac{p_f}{E_c} + \dfrac{p_f}{E_d}}d\Omega_f = \dfrac{p_f}{E}\dfrac{d\Omega_f}{(4\pi)^2}$. Now let us consider the decay of a particle of mass m. With $E = m$, (1.49) gives

$$\Gamma_{a,cd} = \frac{1}{2m}\frac{p_f}{E}\int |M_{a,cd}|^2\frac{d\Omega_f}{(4\pi)^2}. \tag{1.50}$$

By integrating the above equation on the angles, we obtain

$$\Gamma_{a,cd} = \frac{p_f}{8\pi m^2}\overline{|M_{a,cd}|^2} \tag{1.51}$$

where the angular average of the absolute square of the matrix element appears.

Now let us consider the cross section of the process $a + b \to c + d$, in the centre of mass frame. Again let E_a and E_b be the initial energies, E_c and E_d the final ones. The total energy is $E = E_a + E_b = E_c + E_d$. Let $\mathbf{p}_i = \mathbf{p}_a = -\mathbf{p}_b$ be the initial momenta and $\mathbf{p}_f = \mathbf{p}_c = -\mathbf{p}_d$ the final ones.

Let us restrict ourselves to the case in which neither the beam nor the target is polarised and in which the final polarisations are not measured. Therefore, in the evaluation of the cross section we must sum over the final spin states and average over the initial ones. Using (1.48) we have

$$\frac{d\sigma}{d\Omega_f} = \frac{1}{2E_a 2E_b|\boldsymbol{\beta}_a - \boldsymbol{\beta}_b|}\overline{\sum_{\text{initial}}}\sum_{\text{final}}|M_{fi}|^2\frac{1}{(4\pi)^2}\frac{p_f}{E}. \tag{1.52}$$

We evaluate the difference between the speeds

$$|\boldsymbol{\beta}_a - \boldsymbol{\beta}_b| = \beta_a + \beta_b = \frac{p_i}{E_a} + \frac{p_i}{E_b} = \frac{p_i E}{E_a E_b}.$$

Hence

$$\frac{d\sigma}{d\Omega_f} = \frac{1}{(8\pi)^2}\frac{1}{E^2}\frac{p_f}{p_i}\overline{\sum_{\text{initial}}}\sum_{\text{final}}|M_{fi}|^2. \tag{1.53}$$

The average over the initial spin states is the sum over them divided by their number. If s_a and s_b are the spins of the colliding particles, then the spin multiplicities are $2s_a+1$ and $2s_b+1$. Hence

$$\frac{d\sigma}{d\Omega_f}=\frac{1}{(8\pi)^2}\frac{1}{E^2}\frac{p_f}{p_i}\frac{1}{(2s_a+1)(2s_b+1)}\sum_{\text{initial}}\sum_{\text{final}}\left|M_{fi}\right|^2. \tag{1.54}$$

1.7 Hadrons, leptons and quarks

The particles can be classified, depending on their characteristics, into different groups. We shall give here the names of these groups and summarise their properties.

The particles of a given type, the electrons for example, are indistinguishable. Take for example a fast proton hitting a stationary one. After the collision, that we assume to be elastic, there are two protons moving in general in different directions with different energies. It is pointless to try to identify one of these as, say, the incident proton.

First of all, we can distinguish the particles of integer spin, in units $\hbar$ $(0,\hbar,2\hbar,\ldots)$, that follow Bose statistics and are called **bosons** and the semi-integer spin particles $\left(\frac{1}{2}\hbar,\frac{3}{2}\hbar,\frac{5}{2}\hbar,\ldots\right)$ that follow Fermi–Dirac statistics and are called **fermions**. We recall that the wave function of a system of identical bosons is symmetric under the exchange of any pair of them, while the wave function of a system of identical fermions is antisymmetric.

Matter is made up of atoms. Atoms are made of **electrons** and **nuclei** bound by the electromagnetic force, whose quantum is the photon.

The **photons** (from the Greek word *phos* meaning *light*) are massless. Their charge is zero and therefore they do not interact among themselves. Their spin is equal to one; they are bosons.

The **electrons** have negative electric charge and spin 1/2; they are fermions. Their mass is small, $m_e=0.511$ MeV, in comparison with that of the nuclei. As far as we know they do not have any structure, they are elementary.

Nuclei contain most of the mass of the atoms, hence of the matter. They are positively charged and made of protons and neutrons. **Protons** (from *proton* meaning *the first*, in Greek) and **neutrons** have similar masses, slightly less than a GeV. The charge of the proton is positive, opposite and exactly equal to the electron charge; neutrons are globally neutral, but contain charges, as shown, for example, by their non-zero magnetic moment. As anticipated, protons and neutrons are collectively called **nucleons**. Nucleons have spin 1/2; they are fermions. Protons are stable, within the limits of present measurements; the reason is that they have another conserved 'charge' beyond the electric charge, the 'baryonic number', which we shall discuss in Chapter 3.

In 1935, Yukawa formulated a theory of the strong interactions between nucleons (Yukawa 1935). Nucleons are bound in nuclei by the exchange of a zero spin particle, the quantum of the nuclear force. Given the finite range of this force, its mediator must be massive. Given the value of the range, about 10^{-15} m, its mass should be intermediate between the electron and the proton masses; therefore it was called the **meson** (*that which is in the middle*). More specifically, it is the π meson, also called the **pion**. We shall describe its properties in the next chapter. Pions come in three charge states: π^+, π^- and π^0. Unexpectedly, from 1946 onwards, other mesons were discovered in cosmic radiation, the K mesons, which come in two different charge doublets, K^+ and K^0, and their antiparticles, K^- and $\bar{K}^0$.

In the same period other particles were discovered that, like the nucleons, have half-integer spin and baryonic number. They are somewhat more massive than nucleons and are called **baryons** (*that which is heavy or massive*). Notice that nucleons are included in this category.

Baryons and mesons are not point-like; instead they have structure and are composite objects. The components of both of them are the **quarks**. In a first approximation, the baryons are made up of three quarks, the mesons of a quark and an antiquark. Quarks interact via one of the fundamental forces, the strong force, that is mediated by the **gluons** (from *glue*). As we shall see, there are eight different gluons; all are massless and have spin one. Baryons and mesons have a similar structure and are collectively called **hadrons** (*hard*, *strong* in Greek). All hadrons are unstable, with the exception of the lightest one, the proton.

Shooting a beam of electrons or photons at an atom we can free the electrons it contains, provided the beam energy is large enough. Analogously we can break a nucleus into its constituents by bombarding it, for example, with sufficiently energetic protons. The two situations are similar with quantitative, not qualitative, differences: in the first case a few eV are sufficient, in the second several MeV are needed. However, nobody has ever succeeded in breaking a hadron and extracting the quarks, whatever the energy and type of the bombarding particles. We have been forced to conclude that quarks do not exist in a free state; they exist only inside the hadrons. We shall see how the Standard Model explains this property, which is called 'quark confinement'.

The spin of the quarks is 1/2. There are three quarks with electric charge $+2/3$ (in units of the elementary charge), called up-type, and three with charge $-1/3$ called down-type. In order of increasing mass the up-type are: 'up' u, 'charm' c and 'top' t, the down-type are: 'down' d, 'strange' s and 'beauty' b. Nucleons, hence nuclei, are composed of up and down quarks, uud the proton, udd the neutron.

The electrons are also members of a class of particles of similar properties, the **leptons** (*light* in Greek, but there are also heavy leptons). Their spin is 1/2. There are three charged leptons, the electron e, the muon μ and the tau τ, and three neutral leptons, the **neutrinos**, one for each of the charged leptons. The electron is stable, the μ and the τ are unstable, and all the neutrinos are stable.

For every particle there is an antiparticle with the same mass and the same lifetime and all charges of opposite values: the positron for the electron, the antiproton, the antiquarks, etc.

One last consideration: astrophysical and cosmological observations have shown that 'ordinary' matter, baryons and leptons, makes up only a small fraction of the total mass of the Universe, no more than 20%. We do not know what the rest is made of. There is still a lot to understand beyond the Standard Model (see Chapter 10).

1.8 The fundamental interactions

Each of the interactions is characterised by one, or more, 'charge' that, like the electric charge, is the source and the receptor of the interaction. The Standard Model is the theory that describes all the fundamental interactions, except gravitation. For the latter, we do not yet have a microscopic theory, but only a macroscopic approximation, so-called general relativity. We anticipate here that the intensity of the interactions depends on the energy scale of the phenomena under study.

The source and the receptor of the gravitational interaction is the energy-momentum tensor; consequently this interaction is felt by all particles. However, gravity is extremely weak at all the energy scales experimentally accessible and we shall neglect its effects.

Let us find the orders of magnitude by the following dimensional argument. The fundamental constants, the Newton constant G_N of gravity, the speed of light c, the Lorentz transformations, and the Planck constant $\hbar$ of quantum mechanics, can be combined in an expression with the dimensions of mass which is called the Planck mass

$$\begin{aligned} M_\mathrm{P} &= \sqrt{\frac{\hbar c}{G_\mathrm{N}}} = \sqrt{\frac{1.06\times10^{-34}\,\mathrm{J\,s}\times3\times10^{8}\,\mathrm{m\,s^{-1}}}{6.67\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}}} \\ &= 2.18\times10^{-8}\,\mathrm{kg} = 1.22\times10^{19}\,\mathrm{GeV}. \end{aligned} \tag{1.55}$$

It is enormous, not only in comparison to the energy scale of the Nature around us on Earth (eV) but also of nuclear (MeV) and subnuclear (GeV) physics. We shall never be able to build an accelerator to reach such an energy scale. We must search for quantum features of gravity in the violent phenomena naturally occurring in the Universe.

All the known particles have weak interactions, with the exception of photons and gluons. This interaction is responsible for beta decay and for many other types of decays. The weak interaction is mediated by three spin one mesons, W^+, W^- and Z^0; their masses are rather large, in comparison to, say, the proton mass (in round numbers $M_W \approx 80$ GeV, $M_Z \approx 90$ GeV). Their existence becomes evident at energies comparable to those masses.

All charged particles have electromagnetic interactions. This interaction is transmitted by the photon, which is massless. Quarks and gluons have strong interactions; the leptons do not. The corresponding charges are called 'colours'. The interaction amongst quarks in a hadron is confined inside the hadron. If two hadrons, two nucleons for example, come close enough (typically 1 fm) they interact via the 'tails' of the colour field that, shall we say, leaks out of the hadron. The phenomenon is analogous to the van der Waals force that is due to the electromagnetic field leaking out from a molecule. Therefore the nuclear (Yukawa) forces are not fundamental.

As we have said, the charged leptons more massive than the electron are unstable; the lifetime of the muon is about 2 μs, that of the τ, 0.3 ps. These are large values on the scale of elementary particles, characteristic of weak interactions.

All mesons are unstable: the lifetimes of $\pi^\pm$ and of $K^\pm$ are 26 ns and 12 ns respectively; they are weak decays. In the 1960s, other larger mass mesons were discovered; they have strong decays and extremely short lifetimes, of the order of 10^{-23}–10^{-24} s.

All baryons, except for the proton, are unstable. The neutron has a beta decay into a proton with a lifetime of 886 s. This is exceptionally long even for the weak interaction standard because of the very small mass difference between neutrons and protons. Some of the other baryons, the less massive ones, decay weakly with lifetimes of the order of 0.1 ns, others, the more massive ones, have strong decays with lifetimes of 10^{-23}–10^{-24} s.

Example 1.10 Consider an electron and a proton standing at a distance r. Evaluate the ratio between the electrostatic and the gravitational forces. Does it depend on r?

$$F_{\text{electrost.}}(ep) = \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r^2} \qquad F_{\text{gravit.}}(ep) = G_{\text{N}}\frac{m_e m_p}{r^2}.$$

$$\frac{F_{\text{electrost.}}(ep)}{F_{\text{gravit.}}(ep)} = \frac{q_e^2}{4\pi\varepsilon_0 G_{\mathrm{N}} m_e m_p}$$
$$= \frac{(1.6\times 10^{-19})^2}{4\pi\times 8.8\times 10^{-12}\times 6.67\times 10^{-11}\times 9.1\times 10^{-31}\times 1.7\times 10^{-27}} \approx 10^{39}$$

independent of r.

1.9 The passage of radiation through matter

The Standard Model has been developed and tested by a number of experiments, some of which we shall describe. This discussion is not possible without some knowledge of the physics of the passage of radiation through matter, of the main particle detectors and the sources of high-energy particles.

When a high-energy charged particle or a photon passes through matter, it loses energy that excites and ionises the molecules of the material. It is through experimental observation of these alterations of the medium that elementary particles are detected. Experimental physicists have developed a wealth of detectors aimed at measuring different characteristics of the particles (energy, charge, speed, position, etc.). This wide and very interesting field is treated in specialised courses and books. Here we shall only summarise the main conclusions relevant for the experiments we shall discuss in the text and not including, in particular, the most recent developments.

Ionisation loss

The energy loss of a relativistic charged particle more massive than the electron passing through matter is due to its interaction with the atomic electrons. The process results in a trail of ion–electron pairs along the path of the particle. These free charges can be detected. Electrons also lose energy through bremsstrahlung in the Coulomb fields of the nuclei.

The expression of the average energy loss per unit length of charged particles other than electrons is known as the Bethe–Bloch equation (Bethe 1930). We give here an approximate expression, which is enough for our purposes. If z is the charge of the particle, ρ the density of the medium, Z its atomic number and A its atomic mass, the equation is

$$-\frac{dE}{dx} = K\frac{\rho Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{I} - \beta^2\right)\right] \tag{1.56}$$

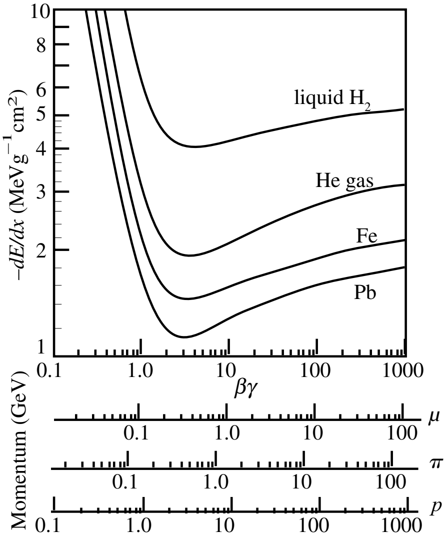

Fig. 1.6. Specific average ionisation loss for relativistic particles of unit charge. (Simplified from Yao *et al.* 2006 by permission of Particle Data Group and the Institute of Physics)

where m is the electron mass (the hit particle), the constant K is given by

$$K = \frac{4\pi\alpha^2(\hbar c)^2 N_A (10^3\,\mathrm{kg})}{mc^2} = 30.7\,\mathrm{keV\,m^2\,kg^{-1}} \tag{1.57}$$

and I is an average ionisation potential. For $Z > 20$ it is approximately $I \approx 12\,Z\,\mathrm{eV}$. The energy loss is a universal function of $\beta\gamma$ in a very rough approximation, but there are important differences in the different media, as shown in Fig. 1.6. The curves are drawn for particles of charge $z = 1$; for larger charges, multiply by z^2.

All the curves decrease rapidly at small momenta (roughly as $1/\beta^2$), reach a shallow minimum for $\beta\gamma = 3$–4 and then increase very slowly. The energy loss of a minimum ionising particle (mip) is (0.1–0.2 MeV $\mathrm{m^2\,kg^{-1}}$)ρ.

The Bethe–Bloch formula is only valid in the energy interval corresponding to approximately $0.05 < \beta\gamma < 500$. At lower momenta, the particle speed is comparable to the speed of the atomic electrons. In these conditions a, possibly large, fraction of the energy loss is due to the excitation of atomic and molecular levels, rather than to ionisation. This fraction must be detected as light, coming from the de-excitation of those levels or, in a crystal, as phonons.

At energies larger than a few hundred GeV for pions or muons, much larger for protons, another type of energy loss becomes more important than ionisation, the bremsstrahlung losses in the nuclear fields. Consequently, dE/dx for muons and pions grows dramatically at energies larger than or around one TeV.

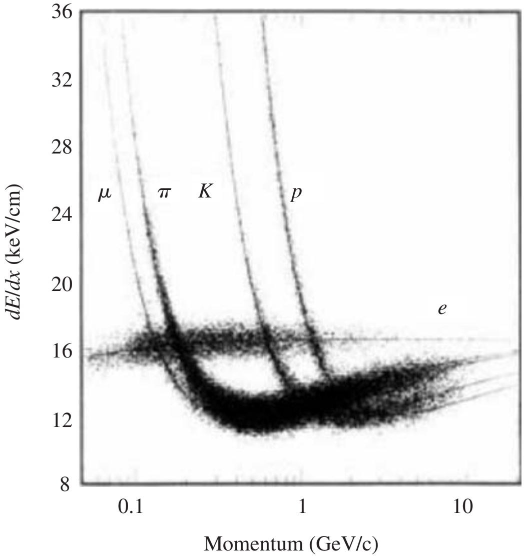

Fig. 1.7. *dE/dx* measured in a TPC at SLAC. (Aihara *et al.* 1988)

Notice that the Bethe–Bloch formula gives the *average* energy loss, while the measured quantity is the energy loss for a given length. The latter is a random variable with a frequency function centred on the expectation-value given by the Bethe–Bloch equation. The variance, called the straggling, is quite large. Figure 1.7 shows a set of measurements of the ionisation losses as functions of the momentum for different particles. Notice, in particular, the dispersion around the average values.

Energy loss of the electrons

Figure 1.7 shows that electrons behave differently from other particles. As anticipated, electrons and positrons, due to their small mass, lose energy not only by ionisation but also by bremsstrahlung in the nuclear Coulomb field. This happens at several MeV.

As we have seen in Example 1.4, the process $e^- \to e^- + \gamma$ cannot take place in vacuum, but can happen near a nucleus. The reaction is

$$e^- + N \to e^- + N + \gamma \tag{1.58}$$

where N is the nucleus. The case of positrons is similar

$$e^+ + N \to e^+ + N + \gamma. \tag{1.59}$$

Classically, the power radiated by an accelerating charge is proportional to the square of its acceleration. In quantum mechanics, the situation is similar: the probability of radiating a photon is proportional to the acceleration squared.

Therefore, this phenomenon is much more important close to a nucleus than to an atomic electron. Furthermore, for a given external field, the probability is inversely proportional to the mass squared. We understand that for the particle immediately more massive than the electron, the muon that is 200 times heavier, the bremsstrahlung loss becomes important at energies larger by four orders of magnitude.

Comparing different materials, the radiation loss is more important if Z is larger. More specifically, the materials are characterised by their radiation length X_0. The radiation length is defined as the distance over which the electron energy decreases to $1/e$ of its initial value due to radiation, namely

$$-\frac{dE}{E} = \frac{dx}{X_0}. \tag{1.60}$$

The radiation length is roughly inversely proportional to Z and hence to the density. A few typical values are: air at n.t.p. $X_0 \approx 300\,\text{m}$; water $X_0 \approx 0.36\,\text{m}$; carbon $X_0 \approx 0.2\,\text{m}$; iron $X_0 \approx 2\,\text{cm}$; lead $X_0 \approx 5.6\,\text{mm}$. We show in Fig. 1.8 the electron energy loss in lead; in other materials the behaviour is similar. At low energies the ionisation loss dominates, at high energies the radiation loss becomes more important. The crossover, when the two losses are equal, is called the critical energy. With a good approximation it is given by

$$E_c = 600\,\text{MeV}/Z. \tag{1.61}$$

For example, the critical energy of lead, which has $Z = 82$, is $E_c = 7$ MeV.

Energy loss of the photons

At energies of the order of dozens of electronvolts, the photons lose energy mainly by the photoelectric effect on atomic electrons. Above a few keV, the Compton effect becomes important. When the production threshold of the electron–positron

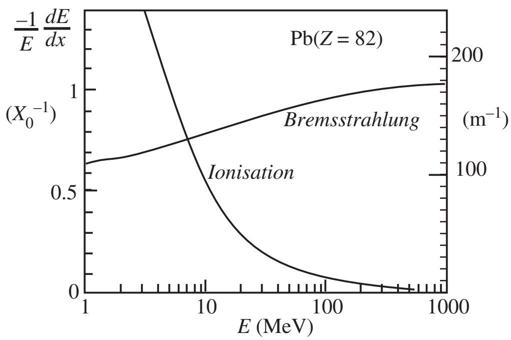

Fig. 1.8. Relative energy loss of electrons in lead. (Adapted from Yao *et al.* 2006 by permission of Particle Data Group and the Institute of Physics)

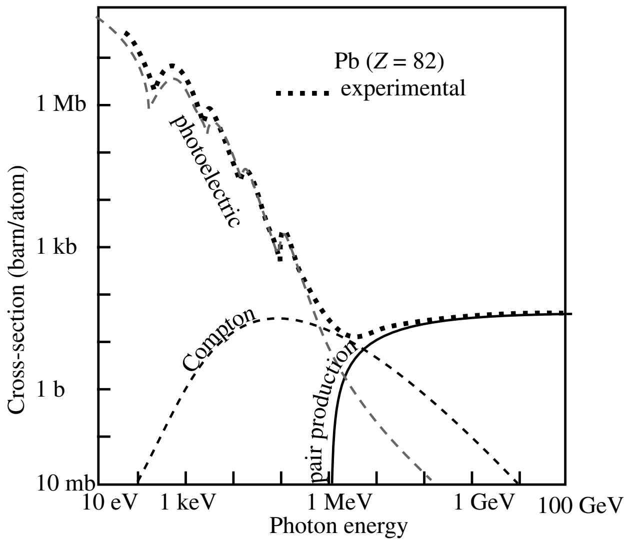

Fig. 1.9. Photon cross sections in Pb versus energy; total and calculated contributions of the three principal processes. (Adapted from Yao *et al.* 2006 by permission of Particle Data Group and the Institute of Physics)

pairs is crossed, at 1.022 MeV, this channel rapidly becomes dominant. The situation is shown in Fig. 1.9 in the case of lead.

In the pair production process

$$\gamma + N \rightarrow N + e^- + e^+ \tag{1.62}$$

a photon disappears, it is absorbed. The attenuation length of the material is defined as the length that attenuates the intensity of a photon beam to $1/e$ of its initial value. The attenuation length is closely related to the radiation length, being equal to $(9/7)X_0$. Therefore, X_0 determines the general characteristics of the propagation of electrons, positrons and photons.

Energy loss of the hadrons

High-energy hadrons passing through matter do not lose energy by ionisation only. Eventually they interact with a nucleus by the strong interaction. This leads to the disappearance of the incoming particle, the production of secondary hadrons and the destruction of the nucleus. At energies larger than several GeV, the total cross sections of different hadrons become equal within a factor of 2 or 3. For example, at 100 GeV the cross sections π^+p, π^-p, π^+n, π^-n are all about 25 mb, those for pp and pn about 40 mb. The collision length λ_0 of a material is defined as the distance over which a neutron beam (particles that do not have electromagnetic interactions) is attenuated by $1/e$ in that material.

Typical values are: air at n.t.p. $\lambda_0 \approx 750\,\mathrm{m}$; water $\lambda_0 \approx 0.85\,\mathrm{m}$; carbon $\lambda_0 \approx 0.38\,\mathrm{m}$; iron $\lambda_0 \approx 0.17\,\mathrm{m}$; lead $\lambda_0 \approx 0.17\,\mathrm{m}$. Comparing with the radiation length we see that collision lengths are larger and do not depend heavily on the material, provided this is solid or liquid. These observations are important in the construction of calorimeters (see Section 1.11).

1.10 Sources of high-energy particles

The instruments needed to study the elementary particles are sources and detectors. We shall give, in both cases, only the pieces of information that are necessary for the following discussions. In this section, we discuss the sources, in the next the detectors.

There is a natural source of high-energy particles, the cosmic rays; the artificial sources are the accelerators and the colliders.

Cosmic rays

In 1912, V. F. Hess, flying aerostatic balloons at high altitudes, discovered that charged particle radiation originated outside the atmosphere, in the cosmos (Hess 1912). Fermi formulated a theory of the acceleration mechanism in 1949 (Fermi 1949). Until the early 1950s, when the first high-energy accelerators were built, cosmic rays were the only source of particles with energy larger than a GeV. The study of cosmic radiation remains, even today, fundamental for both subnuclear physics and astrophysics.

We know rather well the energy spectrum of cosmic rays, which is shown in Fig. 1.10. It extends up to 100 EeV (10^{20} eV), 12 orders of magnitude on the energy scale and 32 orders of magnitude on the flux scale. To make a comparison, notice that the highest-energy accelerator, the LHC at CERN, has a centre of mass energy of 14 TeV, corresponding to 'only' 0.1 EeV. At these extreme energies the flux is very low, typically one particle per square kilometre per century. The Pierre Auger observatory in Argentina has an active surface area of 3000 km^2 and is starting to explore the energy range above EeV. In this region, one may well discover phenomena beyond the Standard Model.

The initial discoveries in particle physics, which we shall discuss in the next chapter, used the spectrum around a few GeV, where the flux is largest, tens of particles per square metre per second. In this region the primary composition, namely at the top of the atmosphere, consists of 85% protons, 12% alpha particles, 1% heavier nuclei and 2% electrons.

A proton or a nucleus penetrating the atmosphere eventually collides with a nucleus of the air. This strong interaction produces pions, less frequently K mesons and, even

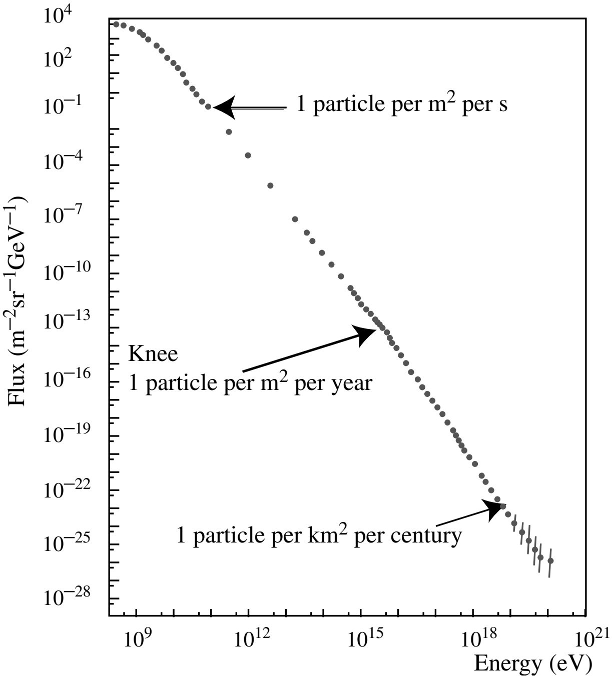

Fig. 1.10. The cosmic ray flux.

more rarely, other hadrons. The hadrons produced in the first collision generally have enough energy to produce other hadrons in a further collision, and so on. The average distance between collisions is the collision length ($\lambda_0 = 750$ m at n.t.p.). The primary particle gives rise to a 'hadronic shower': the number of particles in the shower initially grows, then, when the average energy becomes too small to produce new particles, decreases. This is because the particles of the shower are unstable. The charged pions, which have a lifetime of only 26 ns, decay through the reactions

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \pi^- \to \mu^- + \overline{\nu}_\mu. \tag{1.63}$$

The muons, in turn, decay as

$$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e \qquad \mu^- \to e^- + \nu_\mu + \overline{\nu}_e. \tag{1.64}$$

The muon lifetime is 2 μs, much larger than that of the pions. Therefore, the composition of the shower becomes richer and richer in muons while travelling through the atmosphere.

The hadronic collisions produce not only charged pions but also π^0. These latter decay quickly with the electromagnetic reaction

$$\pi^0 \to \gamma + \gamma. \tag{1.65}$$