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Nonlinear Phenomena in Complex Systems

Edited by

A. N. Proto

**Proceedings of the Workshop on
Nonlinear Phenomena in Complex Systems
Mar del Plata, Argentina, November 1–14, 1988**

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*Laboratorio de Fisica
Comisión Nacional
de Investigaciones Espaciales
Vicente Lopez, Argentina*



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PREFACE

The Workshop on Non-linear Phenomena, and the Third Argentine Workshop on Non-Equilibrium Statistical Mechanics and Non-linear Physics were held in Mar del Plata, a city on the seashore of Buenos Aires Province, from 1–14 November, 1988. The series of Argentine workshops was known as “MEDYFINOL” conferences (a shortened form for Mecánica Estadística del Desequilibrio y Física no Lineal). During 1986, 1987, 1988 (and also in 1989) I had the pleasure of sharing the direction of these meetings with Prof. Angel Plastino (Dept. of Physics, Universidad Nacional de La Plata), Dra. Susana Hernández (Dept. of Physics, Universidad de Buenos Aires), and Dr. Dino Otero (Laboratorio de Física-Matemática, Comisión Nacional de Energía Atómica).

Our main purpose was to provide a forum for discussing recent developments and communicating the latest work of the researchers, as well as to give intensive training to advanced theoretical physics students in the pertinent topics.

This year, our meeting was included in the activities supported by the Office of External Activities of the International Center for Theoretical Physics (ICTP). On behalf of the participants, the conference directors would like to acknowledge, with gratitude, the generous financial support of Professor Abdus Salam, which allowed us to have three distinguished lecturers — John Clark, Rudolf Friederich, and Hans Herrmann. We would also like to acknowledge the valuable support of the Comisión de Investigaciones Científicas de la Provincia de Buenos Aires and the Secretaría de Ciencia y Técnica de la Nación as well as the sponsorship given by the Comisión Nacional de Investigaciones Espaciales, the Universidad de Buenos Aires, the Universidad Nacional de La Plata, the Universidad Tecnológica Nacional, UNESCO, the Latin American Centre of Physics (CLAF), and the Comisión Nacional de Energía Atómica. To the Universidad Nacional de Mar del Plata, the host institution of the meeting, and to the local organizers, Lic. Juan R. Sanchez, Lic. Constancio M. Arizmendi, and Mr. Alberto H. Rizzo, we express our gratitude for their kindness and efficiency.

The success of the conference was partly due to the active support of three members of my research group, Dr. Jorge Aliaga, Lic. Gustavo Crespo, and Lic. Luis Irastorza, who helped with the organization and carried out a wide variety of tasks. They courteously helped me to prepare these proceedings, by gathering, reading, and undertaking the onerous task of typing some of the conference material. To them, my special thanks.

INTRODUCTION

This book provides a thorough treatment of neural networks, cellular-automata, and synergetics, to illustrate three different approaches to non-linear phenomena in complex systems. These topics are of special interest to physicists working in the fields of statistical mechanics and dynamical systems. The chapters are written with a high degree of sophistication and include the refinements necessary to work with the complexity of real systems. Recent research developments in these areas are included as well.

Neural networks are currently arousing the interest of scientists in a number of disciplines: as models for understanding how the brain works; as a way to exploit the principles of natural intelligence for practical purposes; as examples of complex statistical systems; as modern computational structures; and as devices for intelligent pattern recognition. Dissipative non-linear dynamical systems of interconnected neuronal elements are treated here by providing a broad survey of the modelling of neural phenomena.

Cellular automata play a central role in the understanding of complexity and its origins, since traditional mathematical models and methods do not seem to be well suited to study complex systems. Physics', biology's and other fields' experiments that show complex behavior can be successfully approached with these models which are simply constructed, but contain the essential mathematical features that can reproduce experimental data. Cellular automata can also be viewed as computational models and they are likely to be particularly important to the implementation of future generations of parallel computers.

The essence of synergetics lies in the study of the cooperation of the many subsystems which compose a complex system. This cooperation gives rise to spatial, temporal and functional structures. Special attention is given to those systems in which these structures appear in a self-organized fashion. The principles governing these self-organization processes can be studied, irrespective of the nature of the component subsystems. The applications of synergetics ranges from physics to sociology, making the importance of its basic concepts and mathematical approach evident.

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INTRODUCTION TO NEURAL NETWORKS

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Neural networks are currently exciting the interest of scientists across a broad range of disciplines including neurobiology, psychology, computer science, and theoretical physics. Neural nets are viewed as models of neurophysiological and cognitive function, as computational structures, as algorithms for solution of optimization and pattern recognition problems, as computing structures, as novel dynamical systems, as complex statistical systems characterized by disorder and frustration, These notes provide a broad survey of the modeling of neural phenomena, such as memory and learning, in terms of dissipative, nonlinear dynamical systems of interconnected neuronal elements. In general, synaptic interactions between two neurons are not reciprocal. And in general, these interactions may change with time, in a way that depends on the recent activity or experience of the network. Models based on binary threshold elements operating in discrete time are constructed which display emergent computational properties and fascinating dynamical behavior; in particular they show a capacity for distributed, content-addressable memory and may undergo dynamical phase transitions. Models based on frequency coding, and operating in continuous time, are used to mimic EEG activity in the mammalian olfactory bulb and to demonstrate the potential for chaotic activity in neural systems. Models with probabilistic time evolution are introduced to describe the stochastic processes which underlie information transfer at synaptic junctions and to explore unconventional aspects of the nonequilibrium statistical mechanics of neural networks.

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1. ELEMENTS OF NEUROBIOLOGY

1.1 *The challenge of brain science*

Over the centuries, thinkers and scientists have sought an understanding of the workings of the human mind or brain. It is currently fashionable to seek such an understanding in terms of the behavior of a complex, nonlinear dynamical system of many interacting subunits. The brain is indeed a complex system *par excellence*. It contains a number of nerve cells, or neurons, of the order 10^{11} , which interact at some $10^{14} - 10^{15}$ synaptic junctions. There is great diversity in the sizes and shapes of the various neuron types, and a single neuron may have as many as 200,000 synaptic inputs. The interactions between two neurons are in general not reciprocal, so the usual equality of action and reaction encountered in physics does not hold in the neural domain. The pattern of connectivity is extremely intricate, displaying a high degree of specificity in a background of randomness. A quasi-crystalline architecture can be recognized, the neocortex being organized into modular columns of $10^4 - 10^5$ cells with linear dimensions on the millimeter scale. However, the short-range interactions defining this structure are supplemented by a system of connecting fibers which link neurons over long distances. As a result, the minimum number of synaptic junctions which must be traversed by information passing from one neuron in the brain, arbitrarily chosen, to another, arbitrarily chosen, is (on the average) a number not much bigger than 5. Thus, simple models involving nearest-neighbor interactions or homogeneous connectivity, while illuminating, are unrealistic. Moreover, as we move to a subcellular resolution of nervous tissue, on down to the molecular level, there emerges a wealth of new detail which may not be irrelevant to the ultimate description of cognitive phenomena. An understanding of this system will not be won easily.

The first thing that a physicist must appreciate when learning about the problem of mind and brain is that a neural system can be viewed as operating on two different “planes of existence,” or in two different “worlds.” On the one hand, the brain or any part of it is certainly a piece of ordinary matter, with all the properties commonly associated with ordinary matter: temperature, pressure, chemical potentials, electric fields, etc. Thus one may describe the operation of neural matter in the tangible physical universe, determining the nature of chemical reactions and molecular transformations in the synaptic complex, recording the response of neurons to electrical stimulation, tracing the pathways of action potentials in a network of cells, etc. On the other hand, the brain displays a new class of properties not seen at lower levels of organization. These are the emergent collective phenomena associated with cognitive behavior, and belonging to the universe of information processing: memory storage and recall, pattern recognition, feature extraction, association, categorization, generalization, learning, problem solving and purposive behavior, . . . (To this list of objective manifestations one might want to add the subjective – and hence more elusive – phenomena of sensation and consciousness.) In the end it is these remarkable cooperative phenomena which the modern cognitive scientist hopes to understand in the language of the physics of complex systems, modeling the living nerve net as an assembly of interacting elements whose evolution is governed by definite dynamical laws. The attainment of this goal will provide one of the major scientific challenges of the 21st century.

Currently, there is a contagious excitement about neural modeling. The subfield of neural networks has rapidly become one of the most fashionable in theoretical physics. However, this enthusiasm does not stem from any startling breakthroughs in the modeling of real networks in the brain and in the understanding of how the brain works. The overly simplistic models which are now so popular are grossly out of tune with the difficulty of the fundamental psychobiological problem. On the other hand, such models may be of substantial metaphorical value in understanding how cognitive phenomena *might* arise, and they suggest novel ways in which computations may be performed in both natural and artificial systems. Thus, the excitement about neural networks derives largely from the prospects for new computing structures and new algorithms for synthetic intelligence based on parallel processing and distributed representations. Indeed, we are now witnessing a strong resurgence of the Neurobiological Paradigm: the adaptation of principles thought to underlie natural intelligence in the design of machines which perform useful functions on the cognitive plane.

Both of the aspects just touched upon, namely the use of neural networks for computing, and the use of (elaborated) neural-network models in formulating realistic theories of biological nerve nets, exercise some attraction for physicists, although they belong more naturally to the domains of computer science (including artificial intelligence) and theoretical biology. Most of the publications in physics journals have focused on the intriguing collective behavior of simple neural network models, as revealed by the application of modern techniques in statistical physics and dynamical-systems theory. There is a preoccupation with the thermodynamic limit. In studies of equilibrium statistical properties, mean-field theory is used to determine a phase diagram characterizing content-addressable memory capabilities. Penetrating analogies with disordered systems in physics, especially spin glasses, are exploited. Studies of dynamics entertain the possibility of chaotic activity, along with orderly convergence to fixed points or limit cycles.

These notes are intended to acquaint the novice with all three aspects – the biological, the computational, and the (quasi)physical. There is by now a vast, if unsystematic, literature in the diverse field of neural networks. The flavors of the varied efforts are captured in a number of conference-proceedings volumes, omnibus collections, and review articles. Current work on brain modeling and simulation is exemplified in Ref. 1, while Refs. 2,3 offer a panorama of recent studies in collective computation. Short reviews of concepts and applications in computational neuroscience are given by Refs. 4,5. Ref. 6 has become the source-book for the theory and practice of parallel distributed processing, and for its role in cognitive science; a more recent summary of connectionist procedures is provided by Ref. 7. Detailed accounts of what theoretical physicists have learned about the popular models using statistical methods are given in Refs. 8-10. Necessarily and beneficially, there is considerable overlap of the three aspects of neural networks I have identified (and indeed a fourth – cognitive modeling – which belongs naturally to psychology). The rich mixture of ideas, approaches, and goals is seen in such collections as Refs. 11,12. The engineering orientation of much of the present work is evident in Refs. 13,14, which should be consulted for information about advanced developments in algorithms and hardware. Review articles and commentaries at a popular scientific level, or with pedagogic aims, include Refs. 15-20. The educated layman as well as the expert will enjoy reading Ref. 21. A fascinating and well-documented historical survey of the field may be found in Ref. 22. Ref.

23 may be credited with the birth of neural networks, Ref. 24 amounted to their premature burial, and Ref. 25 heralded the second coming. Several monographs or proceedings predating the revival of the field remain quite useful; these are listed as Refs. 26-35.

1.2 *Biological background for neural modeling*

If we want to build neural networks models which embody principles of neurophysiological organization and function, we must first learn something about real neurons and how they interact. The following is a minimal collection of the relevant facts. This sketch comes with a caveat: the extreme complexity of the nervous system and the diversity of its components render simple statements and generalizations both difficult and dangerous. Authoritative treatments of the background biology are available (for example) in Refs. 36-42.

The characteristic distance scale of the neuronal world is the micron $\mu = 10^{-6} \text{ m} = 10^{-4} \text{ cm}$, and the characteristic time scale is the millisecond (ms).

While acknowledging that there is no such thing as a typical neuron, it is ordinarily possible to identify three main parts of a nerve cell, distinguished by their structure (anatomy) and function (physiology). Referring to the neuron in Fig. 1 labeled *i*, we see that there is a *cell body* or *soma*, which contains the nucleus and is responsible for normal metabolic activity. Extending from the cell body there is a collection of fibers called *dendrites*, which may branch repeatedly but remain near the cell body. The whole dendritic complex may resemble the root system of a tree. Some dendrites are covered with smaller projections called spines. The third major part of the neuron is a smooth fiber called the *axon*, which may extend a considerable distance from the cell body before branching. The great variability in vertebrate neuronal anatomy is evidenced by the range in cell-body diameters (roughly 5-100 μ), in axon thicknesses (normally $\sim 1 \mu$ but as large as 20 μ in extreme cases), and in axon lengths (from $\sim 0.1 \text{ mm}$ to $\sim 1 \text{ meter}$ in man). The variety of dendritic patterns is even more impressive.

Neurons may be divided into many different categories (perhaps hundreds), depending on the criteria adopted (anatomical, physiological, ...?) and on how fine one wants to be in making distinctions. A very simple classification scheme, based on gross appearance but more incisively on gross function, will be the most useful for our purposes. We are primarily interested in vertebrate cortical systems. According to Shepherd,⁴⁰ "a cortical system is a region of the central nervous system in which neurons are differentiated into several distinct types, and their cell bodies and cell processes are organized in several nonrepeating layers." Examples of cortical systems are the cerebellum, the hippocampus, the olfactory bulb, the piriform (olfactory) cortex, and the retina. Most prominent in man is the cerebral cortex, or neocortex, including the motor cortex, the somatosensory cortex, the auditory cortex, the visual cortex, and various associative regions. Cortical neurons may be classed as either *principal cells* or *intrinsic cells*. Principal cells tend to be larger and function as final signaling cells, processing information they receive and sending their output to distant locations, either in the same or in a different cortical structure. They also interact locally with other principal neurons and with intrinsic cells. Examples are the pyramid cells of the neocortex (cerebral cortex) and the extremely intricate Purkinje cells of the cerebellum. Intrinsic cells, generally smaller, act as interneurons, transmitting information locally and modifying the outputs of principal cells. Examples of intrinsic neurons are stellate,

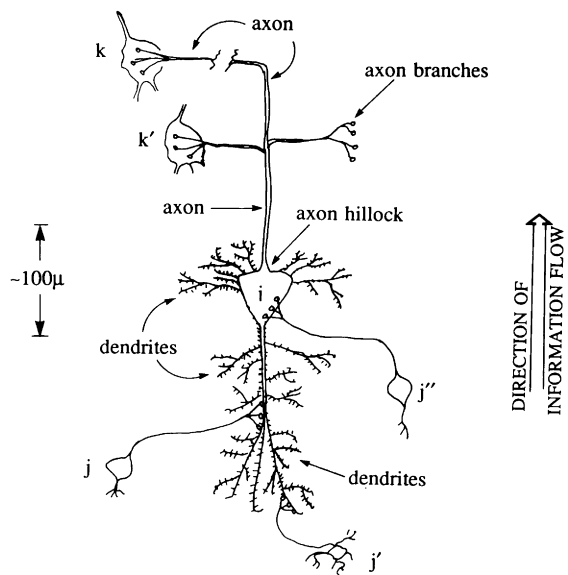


FIGURE 1.

‘Typical’ neuron, showing cell body (soma), dendritic tree, and axon. Note synapses from presynaptic cells j , j' , and j'' onto dendrites and cell body of postsynaptic cell i .

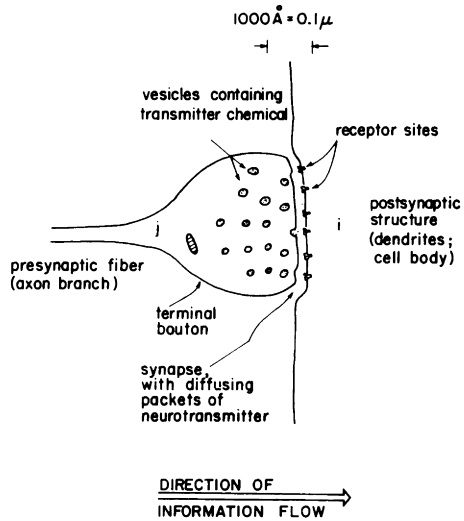


FIGURE 2.

Synaptic junction of terminal of axon branch of cell j onto dendrite or cell body of cell i .