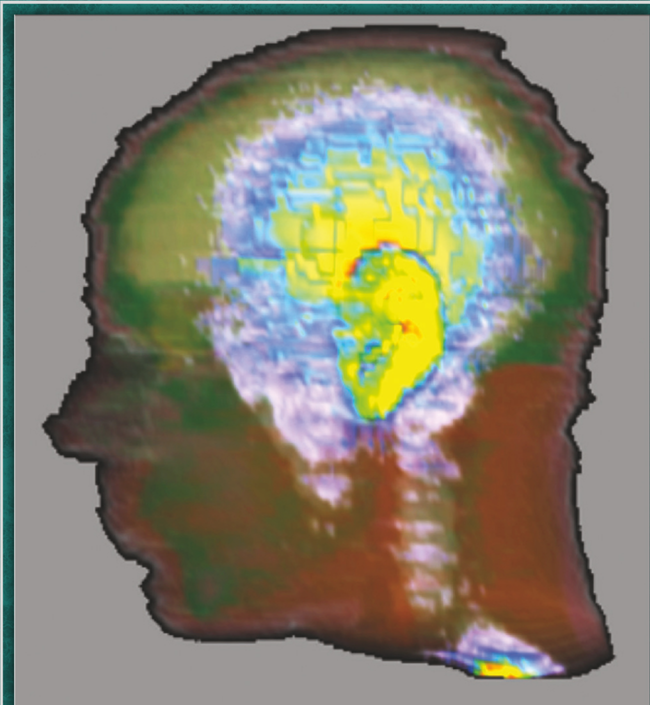
THIRD EDITION

BASIC INTRODUCTION TO BIOELECTROMAGNETICS



JAMES R. NAGEL • CYNTHIA M. FURSE
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For Dad

For Katie

For Laraine

For Marie

Contents

Preface

While doing research in bioelectromagnetics (the interaction of electromagnetic fields with biological systems) for more than 40 years, we have sensed the need some life scientists have to understand the basic concepts and characteristic behaviors of electromagnetic (EM) fields so they can work effectively with physicists and electrical engineers in interdisciplinary research. Because most EM books are based heavily on vector calculus and partial differential equations, however, little written information about EM fields is available to satisfy this need. Many times over the years, life scientists have asked us for references to EM books appropriate for them, but we could give none. These scientists wanted to understand how the fields worked and what controlled them, what factors were important in experimental setups and which were not. They had a great deal of curiosity in how fields were applied in their specific medical application. Yet they rarely, if ever, actually computed the fields themselves. These scientists needed a strong intuitive understanding of electromagnetic fields. We wrote the first edition of this book in an effort to fulfill that need, as well as to help others who want to learn about electromagnetics but do not have the mathematical background to understand typical books on electromagnetics. The second and third editions of this book continue in that vein. The material is rearranged in many places to give the reader the details "just in time" to understand the applications. The second edition is also augmented by over 40 medical applications of electromagnetics commonly found in clinical settings as well as a new and expanded Chapter 6 describing emerging methods and understanding about bioelectromagnetics. These applications are used to illustrate the basic principles in this book and how they are translated into real devices. For the third edition, we brought color to the figures, providing an even more graphical view of the fields and how they are controlled by the biological environment around them. We also updated and added to the medical applications in Chapter 6.

The purpose of this book is to explain the basic concepts, fundamental principles, and characteristic behaviors of electric and magnetic fields to those who do not have a background in vector calculus and partial differential equations. In particular, it is intended for life scientists collaborating with engineers or physicists in work involving the interaction of electromagnetic fields with biological systems. It should also be helpful to health physicists, industrial hygienists, and public health workers concerned with possible hazards or beneficial applications of electromagnetic field exposure and to those concerned with magnetic resonance imaging, implantable medical devices, electrophysiology, optical interactions with tissue, wireless communication devices, and more. Furthermore, this book may also be useful to traditional electrical engineers and physicists who are learning or have already learned the calculus-based mathematical calculations associated with traditional electromagnetics but who would like to have a stronger intuitive grasp of the subject.

In stark contrast to typical EM books that require a background in vector calculus and partial differential equations, this book requires only a background in algebra (some acquaintance with trigonometric functions would also be helpful), but it explains in detail the basic concepts, fundamental principles, and characteristic behaviors of EM fields using pictures, field maps, and graphs and numerous real-world applications. The explanations include a minimum of mathematical relationships, with the emphasis on qualitative behaviors and graphical descriptions. Nevertheless, in spite of the de-emphasis on advanced mathematics, the concepts of EM field theory are still treated in a comprehensive and accurate manner, with probably more intuitive description than would be found in most traditional electromagnetics textbooks. The material covers the entire frequency spectrum from direct current up through optical frequencies. Practical explanations are given to help readers understand real situations involving EM fields. Over 200 illustrations are included to augment qualitative explanations.

The first chapter gives an introduction to the fundamentals of EM field theory and explains how characteristic behaviors can be effectively grouped in three categories defined by the wavelength of the EM fields compared to the size of the objects with which they interact: (1) when the wavelength is much larger than the size of the objects, (2) when it is about the same, and (3) when the wavelength is much smaller than the size of the objects. Chapters 2–4, respectively, explain the characteristic behaviors in each of these three categories and how they are applied to applications in those frequency bands. Chapter 5 explains some of the principles of EM fields that are quantified in detailed and complex environments typical of bioelectromagnetic applications. This calculation of the doses of the electromagnetic fields is called *dosimetry*. The book concludes with Chapter 6, which discusses existing, emerging, and future medical applications of bioelectromagnetics.

We sincerely hope that this book will be useful (and enjoyable!) for its intended readers. We welcome comments and suggestions for improving it.

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Authors

James R. Nagel was born in El Sobrante, California, on November 10, 1980. He received his BS in electrical engineering in 2004, followed by his MS in electrical engineering in 2006, both from Brigham Young University. He then spent the year of 2007 working for Lockheed Martin Technical Operations at Vandenberg Air Force Base, California, before returning to the University of Utah in 2008. James then completed his PhD, also in electrical engineering, in 2011. During that time, he taught numerous courses in applied electromagnetics and numerical methods, earning him the Thomas G. Stockham, Jr Medal for Teaching Excellence in 2012. James worked on several projects for various startup companies, ranging from light extraction with infrared LEDs to diffractive color splitting in liquid crystal displays. In 2015, James finally returned to the University of Utah as a research associate where he now studies eddy current separation of scrap metals and alloys.

Cynthia M. Furse was born in Stillwater, Maine, on May 7, 1963. She received her BSEE degree in 1986, her MSEE in 1988, and her PhD in electrical engineering in 1994, all from the University of Utah. She was an NSF CISE Fellow at the University of Utah from 1994 to 1997, where she developed computational methods for determining the absorption of electromagnetic fields in the head from cellular telephones. She then was an assistant and associate professor of electrical engineering at Utah State University (USU), where she taught electromagnetics, wireless communication, antennas, numerical electromagnetics, microwave engineering, and project management. While at USU, she established the Center of Excellence for Smart Sensors to create and commercialize sensors for evaluating complex environments such as the human body, underground geophysical phenomenon, and wiring systems in aircraft. She was also the director of the Richard and Moonyeen Anderson Wireless Teaching and Research Center. Dr. Furse was the Professor of the Year in the College of Engineering at USU for the year 2000 and the Faculty Employee of the Year 2002. In 2002, she moved to the University of Utah, where she is now a professor of electrical engineering and the Associate Vice President for Research. Dr. Furse's major biological research interests include telemetry systems for the human body, simulation of fields in the body, and coil designs for biological imaging. She is also a founder of LiveWire Innovation, Inc., a university spin-off company commercializing sensors for locating intermittent faults on live aircraft wiring. She has received numerous teaching awards including the Harriett B. Rigas award for educational leadership, Distinguished Educator award in the College of Engineering, and also the Distinguished Young Alumni award in the Department of Electrical and Computer Engineering. She is active in K-12 outreach programs to expose young people to the excitement of engineering.

Dr. Furse is a Fellow of the Institute of Electrical and Electronics Engineers (IEEE), a Fellow of the National Academy of Inventors, a member of Commission K of the Union of Radio Science International, Phi Kappa Phi, Eta Kappa Nu, Tau Beta Bi, the American Society of Engineering Education, the Society of Women Engineers, and the Applied Computational Electromagnetics Society. She is the past-chairman (1999–2007) of the IEEE Antennas and Propagation Society Education Committee and member of the IEEE AP Administrative Committee, and founding member of the editorial board of the *Journal of Smart Structures and Systems*. She has also served as a member of the editorial board of the *IEEE Transactions on Antennas and Propagation*, the *Journal of the Applied Computational Electromagnetics Society*, and the *IEEE Applied Wireless Propagation Letters.*

Carl H. Durney was born in Blackfoot, Idaho, on April 22, 1931. He received a BS degree in electrical engineering from Utah State University in 1958, and MS and PhD degrees in electrical engineering from the University of Utah in 1961 and 1964, respectively.

From 1958 to 1959, he was an associate research engineer with Boeing Airplane Company, Seattle, Washington, where he investigated the use of delay lines in control systems. He has been with the University of Utah since 1963, where he is presently professor emeritus of electrical engineering and professor emeritus of bioengineering. From 1965 to 1966, he worked in the area of microwave avalanche diode oscillators at Bell Telephone Laboratories, Holmdel, New Jersey, while on leave from the University of Utah. He was visiting professor at the Massachusetts Institute of Technology doing research in nuclear magnetic resonance imaging and hyperthermia for cancer therapy during the 1983–1984 academic year while on sabbatical leave from the University of Utah. At the University of Utah, until he retired in 1997, he taught and did research in electromagnetics, engineering pedagogy, electromagnetic biological effects, and medical applications of electromagnetics.

Dr. Durney is or has been a member of IEEE, the Bioelectromagnetics Society, Commissions B and K of the International Union of Radio Science, Sigma Tau, Phi Kappa Phi, Sigma Pi Sigma, Eta Kappa Nu, and the American Society for

Engineering Education (ASEE). He served as vice president (1980–1981) and president (1981–1982) of the Bioelectromagnetics Society, as a member (1979–1988) and chairman (1983–1984) of the IEEE Committee on Man and Radiation, as a member of the American National Standards Institute C95 Subcommittee IV on Radiation Levels and/or Tolerances with Respect to Personnel (1973–1988), as a member of the editorial board of *IEEE Transactions on Microwave Theory and Techniques* (1977–1997), and as a member of the editorial board of *Magnetic Resonance Imaging* (1983–1995). He was a member of the National Council on Radiation Protection and Measurements from 1990 to 1996. He served as a member of the Peer Review Board on Cellular Telephones (Harvard Center for Risk Analysis) from 1994 to 1997. In 1980, Dr. Durney received the Distinguished Research Award, and in 1993, the Distinguished Teaching Award from the University of Utah. In 1982, he received the ASEE Western Electric Fund Award for excellence in teaching, and the Utah Section IEEE Technical Achievement Award. Utah State University named him College of Engineering Distinguished Alumnus in 1983. In 1990, the Utah Engineering Council named him Utah Engineering Educator of the Year. He was elected a fellow of the IEEE in 1992. In 1993, the Bioelectromagnetics Society awarded him the d'Arsonval Medal.

Douglas A. Christensen was born in Bakersfield, California, on December 14, 1939. He attended Brigham Young University in Provo, Utah, graduating with a BS degree in electrical engineering in 1962. He was valedictorian of the College of Engineering. He attended Stanford University in Palo Alto, California, graduating with an MS degree in electrical engineering in 1963. He then pursued a PhD degree in electrical engineering at the University of Utah, Salt Lake City, graduating in 1967. He was awarded a special postdoctoral fellowship from the National Institutes of Health for studying bioengineering, which he took at the University of Washington, Seattle, from 1972 to 1974. In addition, he has pursued research at the University of California at Santa Barbara and at Cornell University, Ithaca, New York.

Dr. Christensen was appointed an assistant professor of electrical engineering at the University of Utah in 1971. He also received an appointment as an assistant professor of bioengineering at the University of Utah in 1974. He was chairman of the Bioengineering Department from 1985 to 1988. He is currently a professor in both departments.

His industrial experience includes Bell Telephone Laboratories, Murray Hill, New Jersey; International Business Machines Corporation, San Jose, California; Hewlett-Packard Company, Palo Alto, California; and General Motors Research Laboratories, Santa Barbara, California. He has also been a consultant for several companies. His research interests range from electromagnetics to optics to ultrasound. He did early work on a fiberoptic temperature probe used for monitoring temperature during electromagnetic hyperthermia and has worked in numerical techniques for electromagnetic applications, mainly using the finite-difference time-domain method, including its use in optics. He authored a textbook titled *Ultrasonic Bioinstrumentation* and has been co-director of the Center of Excellence for Raman Technology at the University of Utah. He has received the Outstanding Teaching Award and the Outstanding Patent Award from the College of Engineering. His recent interests have been in biomedical optics, especially for sensing and imaging applications.

Chapter 1 Electric and magnetic fields

Basic concepts

1.1 Introduction

Bioelectromagnetics—the study of how electric and magnetic fields interact with the body—is a tremendously exciting field. Electromagnetic (EM) fields are all around us: radio and television signals, cellular telephones, fields from power lines and electrical appliances, radar, and more. They are even within our bodies in the endogenous fields that keep our hearts beating, brains thinking, and muscles moving. EM fields can be used to see inside of us to diagnose illness, sometimes before we feel it ourselves, in the form of medical imaging, electrocardiography, electroencephalography, and electrophysiological evaluations. They can heal us through therapeutic interventions for cancer, pain control, bone growth, soft tissue repair, electrophysiological stimulation, and more. And they can injure or kill us through lightning strikes, deep electrical burns, and shock.

EM fields are already used in numerous medical devices, and the future (read more in Chapter 6) promises ever more detailed and localized diagnostic and treatment methods. EM fields may soon help repair or replace damaged nerve pathways. Already, they help the blind to see, the deaf to hear, and the paralyzed to walk again. The promise of bioelectromagnetics seems limited only by our imaginations. However, the promise of bioelectromagnetics is very much limited by the physical nature of the fields themselves and how they can be made to interact with the body. The purpose of this book is to help you understand EM fields and how they interact with the body, how they are created, how they can be measured and evaluated, and how they can be controlled.

This book begins with the field of classical electromagnetics, which stems from the phenomenon that electric charges exert forces on each other. The concepts of electric and magnetic fields are used to describe the multitude of complex bioeffects that result from this basic phenomenon. Although classical EM field theory is typically couched in vector calculus and partial differential equations, many of the basic concepts and characteristic behaviors can be understood without a strong mathematical background. The purpose of this book is to describe and explain these basic concepts and characteristic behaviors with a minimum of mathematics and to show how they are used in a wide variety of bioelectromagnetic applications. In this chapter, we explain the basic concepts of electric and magnetic fields as a basis for what follows in the remainder of the book.

1.2 Electric field concepts

A fundamental law, Coulomb's law, states that electric charges exert forces on each other in a direction along the line between the charges. Charges with the same sign repel, and charges with opposite signs attract, as shown in Figure 1.1(a). The magnitude of the force exerted on one charge by another charge decreases as the square of the distance between the two charges. When there are many charges in a system (which is usually the case in practice), the forces from each charge add up, in their respective directions. Keeping track of all of these individual forces is not reasonable in practice, so the concept of electric field is used to account for these combined forces.

The concept of electric field is illustrated by this thought experiment: Place a small test charge Q_{test} at a point in space P, as shown in Figure 1.1(a). Whatever other charges exist will exert a force on this test charge. Measure that force, denoted by **F**. By definition, the *electric field strength* at point P is given by

$$\mathbf{E} = \mathbf{F}/Q_{test} \text{ (V/m).} \tag{1.1}$$

The direction of **E** is in the direction of the force exerted on the positive test charge Q_{test}. The force on a negative test charge, such as an electron, would be in the opposite direction. The electric field is shown in Figure 1.1(b). The arrows show the direction of the electric field, and the colors represent its strength at each location. Thus, **E** is a force per unit charge. **E** is also called *electric field intensity*, or often just electric field. The units of **E** are volts per meter (V/m).

Because **F** is a vector, **E** is also a vector. A vector is a quantity having both a direction and a magnitude. In this book, vectors are denoted by boldface symbols.

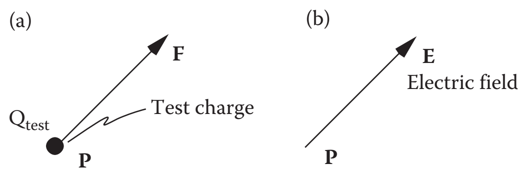

Figure 1.1 (a) A test charge Q_{test} placed at a point P in space is attracted or repelled by other existing charges with force **F**. The direction of the force indicates repelling or attracting forces. The electric field **E** at point P is defined as $\mathbf{E} = \mathbf{F}/Q_{test}$. The direction of the electric field follows the direction of the force. (b) The electric field lines are shown emanating from the charges.

The direction of a vector is represented by an arrow, as in Figure 1.1. The magnitude of a vector is represented by the same symbol as the vector but without boldface. For example, let us define a vector **v** as the velocity having a direction from south to north and a magnitude of 30 meters per second (m/s). Then the magnitude of **v** is expressed as $v = 30\,\text{m/s}$. In a similar fashion, E is the magnitude of the vector **E**, which tells us how strong the field is but not which direction it is in. Figure 1.1(b) shows both the directions of the fields (with arrows) and their magnitudes.

As a consequence of the definition of electric field, a charge Q placed in an electric field **E** will experience a force given by $\mathbf{F} = Q\mathbf{E}$. The larger the **E**, the larger the force **F** exerted on the charge Q. The fundamental effect of an electric field on an object placed in it is to exert forces on the charges in that object, as explained in Section 1.6.

Electric fields are represented graphically in two ways. Figure 1.2 illustrates the first method, using the electric field produced by a single point charge Q, which is perhaps the simplest example of an **E** field. The electric field **E** produced by charge Q will point in the direction a positive test charge would be repelled away from Q, as shown in Figure 1.2. In this first method of displaying **E** fields, the direction of **E** is shown by arrows (pointing away from Q), and the magnitude of **E** is indicated by the closeness of the arrows. In areas where the arrows are close together, the magnitude is higher than in areas where the arrows are farther apart. For example, near the charge Q, the test charge would experience more force and hence electric field than when it is farther away. So, near charge Q, the arrows are close together, indicating a large E. Farther away from the charge, the arrows are farther apart, indicating a smaller E.

The second method of representing vector fields such as **E** is illustrated in Figure 1.3, which shows the **E** field produced by two uniform sheets of charge. In this method, the direction of the **E** field is also shown by arrows. The magnitude of **E** is indicated by the length of the arrows and/or by color. The longer the arrow, or the hotter (redder) the color, the larger the E. This second method is often used when the **E** fields are calculated by numerical methods and plotted by computer

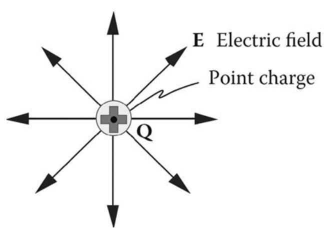

Figure 1.2 Plot of the electric field produced by a single point charge Q.

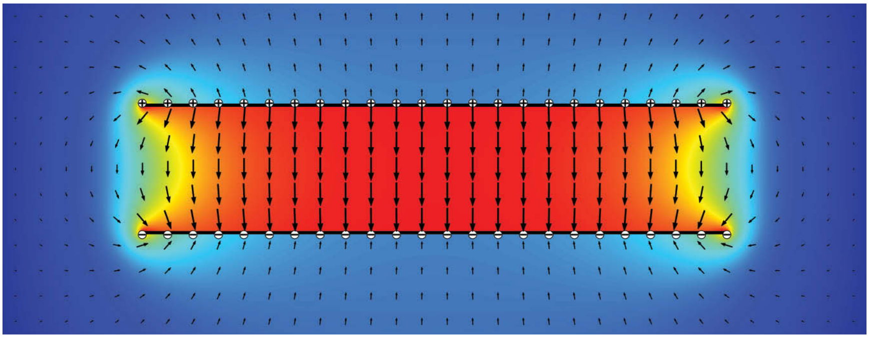

Figure 1.3 **E** field produced by two uniform sheets of charge with positive charge on the top and negative charge at the bottom. The colors represent the magnitude of the **E** field, with red indicating the most intense and blue indicating the least intense. The arrows also indicate both magnitude and direction of the **E** field between the two plates; the **E** field is uniform, with nonuniform fringing fields seen only at the edges.

graphical methods; this is the method we use most often in this book. The **E** field produced by the two uniform sheets of charge is uniform near the center of the sheets. At the edges of the sheets, the **E** bends around or *fringes*.

Because **E** fields exert forces on charges, work is required to move a charge from one point in space to another in the presence of an **E** field. The work done per unit charge is called *electric potential difference*. Electric potential difference is often referred to as potential difference or just *voltage*, because its unit is the volt (V). When **E** is known as a function of space, the potential difference between any two points can be calculated. Let us consider first the simplest case, when **E** is uniform in the space between two points, and a positive charge is moved from one point to another along a path in the opposite direction of **E**, such as moving a charge from point a to point b in Figure 1.4. For this case, the potential difference of point b with respect to point a is given by

$$V_b - V_a = V_{ba} = Ed \ \text{(V)} \tag{1.2}$$

where d is the distance between the two points. Electrical potential difference refers to potential energy. If a charge were moved from point a to point b, it would possess potential energy because if it were released, the force produced on it by **E** would cause it to move, thus converting its potential energy to kinetic energy. When the **E** field is not uniform, or when the path between a and b is not exactly in the opposite direction of **E**, Eq. (1.2) does not apply, and a more complicated calculation is required. Familiar devices such as 12-volt automobile batteries and

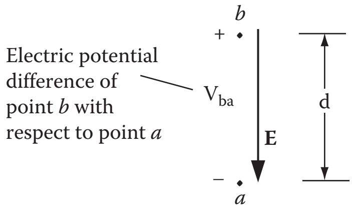

Figure 1.4 Configuration for calculating the potential difference of point *b* with respect to point *a* in the presence of **E**. A positive charge would move from *b* to *a*.

1.5-volt dry cells are used to produce potential differences. Large electric generators produce the potential differences that we use for a multitude of purposes in our homes. Electrocardiograms measure potential differences on the surface of the body caused by the beating heart. External defibrillators create a potential difference between the two paddle electrodes on the body surface. The tissues and fluids in the torso conduct this potential difference to the surface of the heart, where it will hopefully restart the heart.

When **E** does not vary with time, or when it varies slowly with time (the frequency is low), the work done in moving charge between two points is independent of the path over which charge is moved between the two points. In this case, the **E** field is said to be *conservative*, and the potential difference is a unique quantity. When **E** varies rapidly with time (the frequency is high), the work done in moving the charge between two points generally depends upon the path over which the charge is moved between the two points, and a unique potential difference cannot be defined. In this case, **E** is not a conservative field. In special cases (see Section 3.5.1), **E** can vary rapidly with time and still be a conservative field.

Moving charges produce *electric current*, which is defined as the time rate of change of charge. The unit of charge is the coulomb (C). Current at a given point in space is the amount of charge passing that point per second. The unit of current is the ampere (A). Thus, 1 A is equivalent to 1 C/s. *Current density* is defined as current per unit area. Its units are amperes per square meter (A/m^2).

If a time-constant potential difference V is applied between two points and a total current I flows between the two points as a result of this applied voltage, then the current is given by $I = V/R$, where R is the *resistance* (units are ohms) between the two points. As its name implies, resistance opposes the flow of current. This relationship is called *Ohm's law*. It is one of the fundamental laws of electric circuit theory.

The electric field shown in Figure 1.3 could also be produced by replacing the two sheets of charge with metal plates and applying a potential difference between the two, by connecting, for example, a battery between the plates. The potential difference would produce current through the battery, transferring charge from one plate to the other, thus producing charged plates that would be equivalent to the configuration shown in Figure 1.3.

How are electric fields measured?

Electric fields are measured using metallic antennas. Electric fields (e.g., the open lines that travel from positive to negative charges) are picked up by straight antennas, which are oriented parallel to the electric field lines. These straight antennas have a space in the middle that is left open to create a measurable voltage difference. An example is shown in Figure 1.5. This miniature electric field probe antenna was designed for assessment of compliance of EM devices with radio frequency (RF) exposure guidelines. Measurement of fields in or near the body is difficult because a metal object (such as a measurement antenna) can perturb the fields. This small dipole antenna was specifically designed to receive the localized fields without perturbing them. This probe picks up electric fields along its axis, but fields oriented in any other direction are ignored. When all three components of the electric field vector are desired (either separately or in combination to find total electric field strength), three perpendicular linear antennas are used, as shown in the probe in Figure 1.6. Each antenna picks up the electric field parallel to its major axis. The three perpendicular electric field vectors can be measured independently or combined to give total electric field.

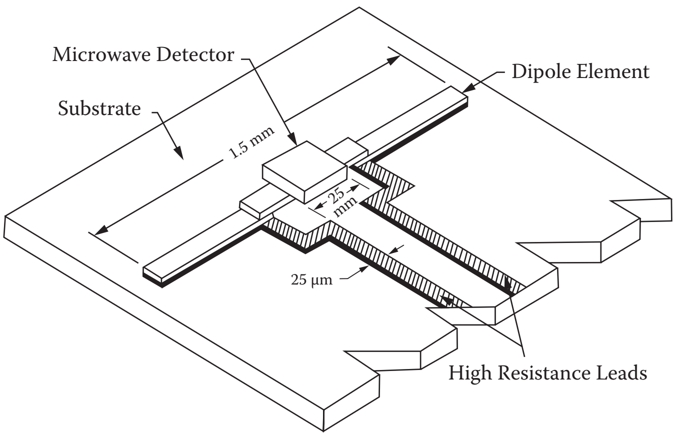

Figure 1.5 Miniature printed dipole antenna for measurement of electric fields to determine cell phone RF exposure compliance. (From Bassen, H., and Smith, G., *IEEE Trans. AP*, 31, 710–718, 1983. © 1983 IEEE. With permission.)

Figure 1.6 Electric field probe made up of three perpendicular antennas. The diameter of the tip is 3.9 mm. (From Schmid & Partner Engineering AG, Zurich. Reprinted with permission.)

1.3 Magnetic field concepts

In the previous section, electric field concepts were explained as a means of accounting for the forces between charges that act on a line between the charges. When charges are moving, they exert another kind of force on each other that is not along a line between the charges. Magnetic fields are used to account for this other kind of force. Moving charges produce an electric current (**I**), shown in the direction of the thumb in Figure 1.7. This current **I** produces a magnetic field **B** in the direction of the fingers in Figure 1.7. The rule that describes the direction of the current and its associated magnetic field is called the *right-hand rule*, because of the use of the right hand to describe it. This rule can be used in two ways. First, the thumb can point in the direction of the current, and the fingers represent the magnetic field (as shown). Alternatively, the thumb can point in the direction of the magnetic field, and the fingers will represent the direction of the current that produced it. The relationships between magnetic field and current are

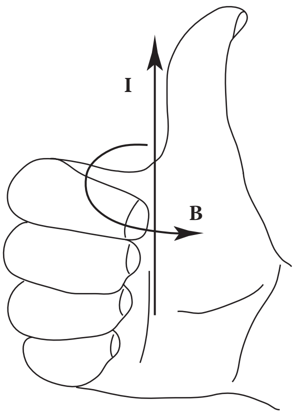

Figure 1.7 The right-hand rule can be used to describe the direction of the current and magnetic field. This rule can be used in two ways. First, the thumb can point in the direction of the current (**I**), and the fingers represent the magnetic (**B**) field (as shown). Alternatively, the thumb can point in the direction of the magnetic field, and the fingers will represent the direction of the current that produced it.

given by Faraday's law. The fact that the magnetic field encircles the current will be discussed in more detail in Section 1.5.

The magnetic field does not produce a force on a stationary charge (like the electric field does), but it does produce a force on any charge that is moving (in addition to that produced by the electric field). The force on a charge Q_{test} moving at a velocity $\mathbf{v}$ at a point P in space is illustrated in Figure 1.8(a). The force on the moving charge has a magnitude of $F = BvQ_{\text{test}}$, where **B** is the *magnetic flux density*. The direction of the force is perpendicular to both **v** and **B**, as shown in Figure 1.8(b). The unit* of **B** is the tesla (T). Magnetic flux density is sometimes referred to as just *magnetic field*.†

Figure 1.9 shows vector plots of the **B** produced by a line current (an infinitely long current coming out of the page) and by a loop current also coming out of (left)/going into (right) the page. The **B** produced by the line current is strongest near the current, as indicated by closer spacing of the arrows. In each case, the **B** lines encircle the current, which is a characteristic described in more detail in Section 1.5.

* A tesla is equivalent to an ampere-henry per square meter. Ampere is the unit of current. Henry is the unit of inductance.

† The related quantity **H** (see Section 1.7) is also often called *magnetic field*. The context is used to keep the meaning clear.

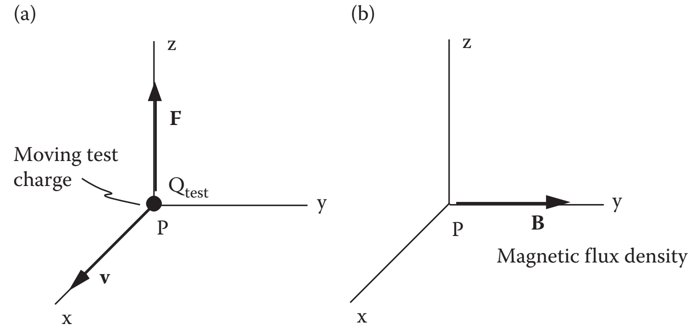

Figure 1.8 (a) Force **F** exerted by a magnetic field on a test charge having velocity **v** at a point *P* in space. **F** is perpendicular to **v**. (b) Magnetic flux density **B** defined at point *P* to account for **F**. **B** is perpendicular to both **v** and **F**.

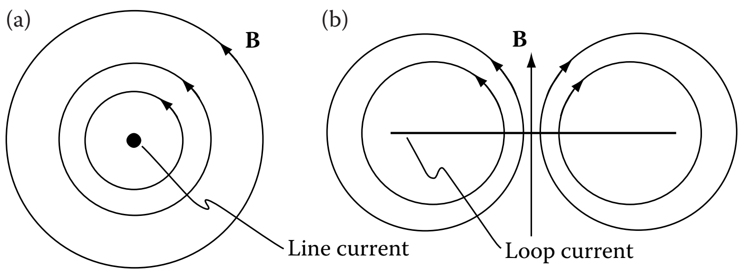

Figure 1.9 (a) **B** fields produced by a line current flowing out of the page. (b) Cross-sectional view of the **B** fields produced by a loop of current oriented perpendicular to the page. The dot indicates current flow out of the page. (An x would have indicated flow into the page.)

How are magnetic fields measured?

Magnetic fields are picked up using loops of wire, and then measuring the induced voltage across the ends of the wire (as discussed in the Section 1.4). The loop may be single or may be a coil of multiple loops, with the loop oriented so that the magnetic field lines pass through the loop. A typical configuration on a commercial magnetic field probe is shown in Figure 1.10. As with the electric field, three separate perpendicular loops can be used to pick up the three components of the magnetic field, as shown in Figure 1.11.

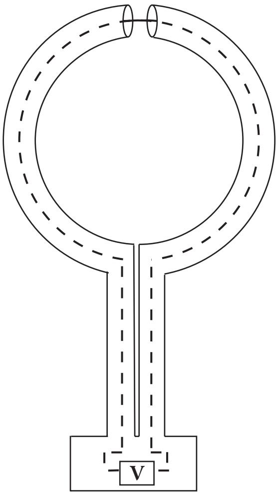

Figure 1.10 Loop antenna used for measuring magnetic fields. The antenna is made from two semirigid coaxial cables. (From Furse, C., et al., *Modern Antennas*, Wiley-Liss, Inc., a subsidiary of John Wiley & Sons, Inc., © Wiley-Liss 2007. With permission.)

Figure 1.11 Magnetic field probes made up of three perpendicular loops. Tip diameter is 6 mm. (From Schmid & Partner Engineering AG, Zurich. Reprinted with permission.)

1.4 Sources of electric fields (Maxwell's equations)

Because **E** fields are defined to account for the forces exerted by charges on each other, the fundamental sources of **E** fields are electric charges. Specific information about how charges act as sources for **E** fields is given by Maxwell's equations, which are a fundamental set of equations that form the framework of all of classical EM field theory. Although we are minimizing the mathematical content of this book, we do state Maxwell's equations below because they are so fundamental and so famous in electromagnetics that we feel you should be introduced to them, even if you may not have a background in vector calculus and partial differential equations. We will just explain the qualitative meaning of these equations without giving the mathematical details.

Two of Maxwell's equations describe sources of **E**. One source is a time-varying **B** field, and the other is charge density ρ. Each source produces **E** fields with specific characteristics. For clarity, we describe these when each source is acting alone, but in general, the **E** is produced by a combination of sources.

The first of Maxwell's equations that we discuss is Faraday's law:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \tag{1.3}$$

$\nabla \times \mathbf{E}$ is a mathematical expression called the *curl* of **E**, which means that the **E** produced will encircle or curl around the **B** that produced it. $\partial \mathbf{B}/\partial t$ is the time rate of change* of **B** (how fast **B** changes, its frequency). This equation tells us that a time-varying magnetic field **B** creates an electric field **E** that curls around it. Generally speaking, the greater the time rate of change of **B**, the stronger **E** field it produces. This new **E** will also be time varying. There are many sources of time-varying magnetic field. Anything that uses typical commercial power (plugs into the wall) has 60-Hz fields.† This means that the fields vary sinusoidally (rise and fall) 60 times per second (see Section 1.9). Fields that vary sinusoidally with time are called *alternating current* (AC) fields. 60 hertz is a relatively slow change in a magnetic field, and therefore the electric field produced is quite small. Generally, we approximate these fields as being constant with time. Fields that do not vary with time (such as those produced by a battery or permanent magnet) are called *direct current* (DC) fields. In the DC case, the magnetic field does not change with time and does not produce an electric field, and we say the fields are *decoupled*. Faster changes in magnetic fields are created in communication systems such as cellular telephones, which operate at 1800–1900 MHz.‡ These sources are also sinusoidal, and the time derivative of the magnetic field is on the order of 10^9 higher than for the 60-Hz commercial power case. Thus, the time-varying magnetic field

* $\partial \mathbf{B}/\partial t$ is the time derivative of **B**. The symbol ∂ means change. So $\partial \mathbf{B}/\partial t$ means change in **B** ($\partial \mathbf{B}$) per change in t (∂t).

† 60 hertz is the standard power frequency in the United States. 50 hertz is used in Europe and Asia.

‡ 1 MHz = 10^6 Hz = 1 million Hz.

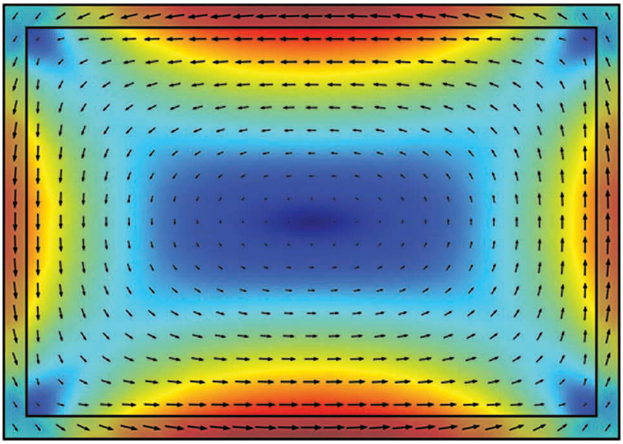

Figure 1.12 Calculated **E** fields at one instant of time for a two-dimensional model consisting of a 1-kHz (1000 Hz) **B** field (directed out of the paper) applied to a nonmetallic container of saline (which is mildly conductive). The electric fields "curl" around the magnetic field.

generates a significant electric field. Other applications utilize fields that are pulsed, such as many imaging applications. For example, some types of microwave tomography use pulses containing frequencies from 300 to 3000 MHz, and a new type of microwave breast imaging called confocal imaging uses pulses with frequencies up to 5000 MHz. These sources are not sinusoidal. They are bursts of energy called *ultrawideband* pulses, and they rise and fall very quickly (microseconds (10^{-6} s) to nanoseconds (10^{-9} s)). Thus, their time rate of change is very high, and a significant electric field is created from the time-varying magnetic field.

Figure 1.12 shows an example of the **E** fields in a nonmetallic container of saline (which is mildly conductive) produced by a changing **B** as calculated from a two-dimensional model.* The **E** field lines encircle (curl around) the changing **B**, which is directed out of the paper. Figure 1.13 shows the same configuration with an object added to the saline that has a higher conductivity (see Section 1.6) than the saline. Here again, the **E** field lines tend to encircle the changing **B**, but they are modified by the presence of the small object having higher conductivity. The higher conductivity of the small object causes the **E** fields inside the object to be weaker than those in the saline. The **E** field pattern in the small object can be thought of as consisting of two components: (1) the globally circulating **E** field of Figure 1.12 without the small object and (2) an **E** field component circulating locally around the center of the small object. The resulting net pattern is a combination of the two, as shown in the magnified view of the object in Figure 1.14. On the left side and near

* A two-dimensional model is constant or equal in the third dimension, in and out of the page.

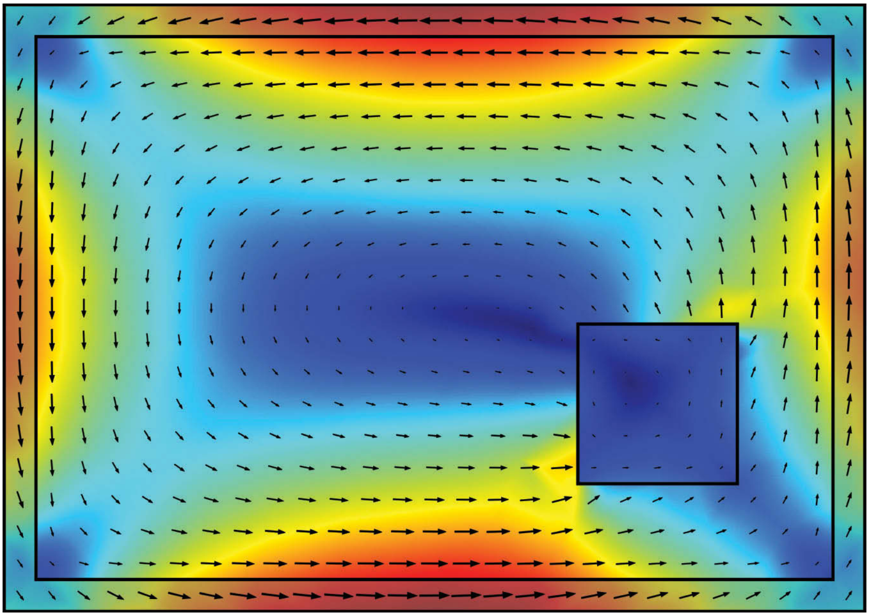

Figure 1.13 The same configuration as in Figure 1.12, but with an object of higher conductivity placed in saline. The electric fields are smaller in the higher-conductivity object. The electric fields would also be smaller if the object had the same conductivity but higher permittivity than the saline.

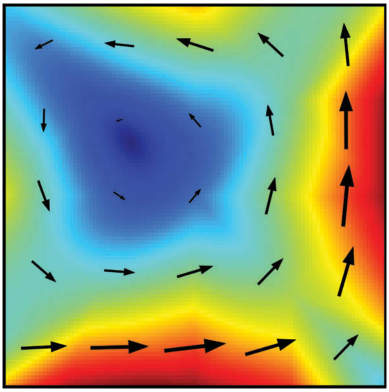

Figure 1.14 A magnified view of the **E** fields in the small object of higher conductivity of Figure 1.13. On the left side and near the top of the object, the globally circulating **E** tends to cancel with the locally circulating **E**, while on the right side and near the bottom of the object, the two fields tend to add, producing a circulating pattern offset from the center of the object.

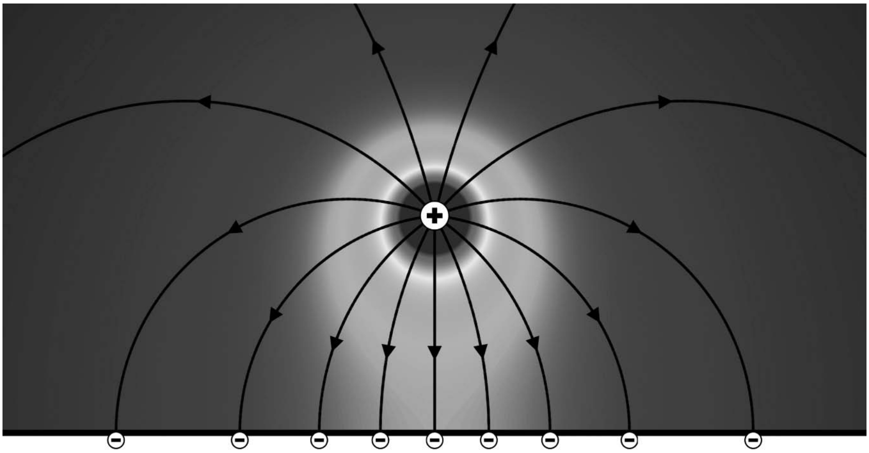

Figure 1.15 A potential difference is applied between a wire (going in/out of the page) and a metal plate (at the bottom). This creates positive charges on the wire and negative charges on the plate. These charges create electric fields going from the positive to negative charges as shown.

the top of the object, the globally circulating **E** tends to cancel with the locally circulating **E**, while on the right side and near the bottom of the object, the two fields tend to add, producing a circulating pattern offset from the center of the object.

A second of Maxwell's equations, Gauss' law, describes the **E** produced by charge density:

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon. \tag{1.4}$$

The expression $\nabla.\ \mathbf{E}$ is called the divergence of **E**, which means an **E** field (arrow) is created that starts at the source, ρ, which is the electric charge density in Coulombs per cubic meter (C/m^3). ε is a parameter called *permittivity*, or *dielectric constant* (see Section 1.6), that just changes the magnitude of the electric field but does not create it or change its direction. Equation (1.4) means that electric charge creates **E**, and that the **E** lines begin and end on charges. For example, this can be seen in Figure 1.3.

Figure 1.15 shows an example of the **E** fields produced by charges. A potential difference applied between a long wire and a metal plate produces positive charges on the wire and negative charges on the plate. These charges produce the kind of **E** field lines shown.

1.5 Sources of magnetic fields (Maxwell's equations)

Another two of Maxwell's equations describe sources of **B**. Ampere's law states that